Adult Numeracy and Workplace Learning: Unpublished Literature Review [2004]

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Introduction

Discussion of adult numeracy and workplace learning can be approached from three perspectives. In the first, following Bernstein (2000), institutional perspectives are addressed, and these concern the processes of new knowledge creation and are connected to knowledge distribution. For the purposes of this review, distinctions between the institution of mathematics and the practices of numeracy in relation to the workplace are addressed. It is at this level that major policy decisions such as the implementation of training packages are formulated, encompassing decisions about who is entitled to learn what. In the second perspective, decisions are made about how these relevant knowledges and skills will be recontextualised into an educational format and how they will be transmitted pedagogically to learners. This review will address issues related to workplace learning in relation to numeracy education. In the third, the processes of acquisition by learners are the focus, and their attempts at reproducing the intended knowledges and skills are evaluated. In the case of workplace numeracy, this is the focus of the research project associated with this review. One of the major issues is the distinction between learning mathematics as the object of an activity (as in school) and the use of numeracy as a tool in the workplace and elsewhere.

This literature review will address the questions:

What is numeracy?

What are the distinctions between mathematics and numeracy?

How is mathematics/numeracy used in the workplace?

What are the curricular and pedagogical implications for workplace education?

Mathematics and Numeracy

What is Numeracy?

There is a swell of voices calling for Australia to become a knowledge economy, to make the most of its human capital. One significant area to emerge in recent years is research into literacy and numeracy practices in restructuring workplaces, especially in relation to automation, emerging communications technologies, and new approaches to workplace organisation and management (FitzSimons 2002). Accordingly, the Federal Government is committed to a policy of encouraging adult numeracy, as well as technological and other literacies, in adult/vocational education.

Key international theoretical debates about the nature and function of generic skills have been influential in Australia (e.g., Dawe 2002; Kearns 2001). However, in recent reviews of numeracy published by the National Centre for Vocational Education Research [NCVER], there are no clear definitions of what is meant by numeracy. However it may be defined, it cannot be assumed that (potential) workers enrolled in Australian VET courses have high levels of numeracy — often due to reasons beyond their control — so there is an urgent need to continue their education. But first, understandings of the term need to be teased out to form a solid basis for curriculum and pedagogy, whether in face-to-face or online situations.

From its nomenclature the term *numeracy* is suggestive of both literacy and numbers, perhaps literacy with numbers; and indeed it is widely accepted that it was coined as a counterpart to literacy. However, over the years its conceptualisation has broadened from being concerned with numbers alone — that is, the 'basics' of the four operations on whole, then rational numbers — to include, in some cases, aspects of algebraic, geometric, statistical thinking (or quantitative literacy), as well as problem solving.

Kanes (2003, p.84) develops a thematic approach to numerical practices: "the theme of *visibility* is about how we formalise and control numerical knowledge; the theme of *useability* is about its use; and the theme of *constructibility* is about its origins both as a cultural-historical phenomenon and as an individual attainment" [italics added]. These three themes could be said to correspond to Bernstein's (2000) fields of knowledge production, recontextualisation, and reproduction respectively; and involve the processes of knowledge creation, transmission, and acquisition.

FitzSimons, Jungwirth, Maasz, and Schlöglmann (1996) trace a brief history of the development of official political interest in this topic, predominantly in Anglophile countries. For example, in the UK, the USA, and Australia there has been a burgeoning public interest, and this has more recently spread to some countries of the European Union (DEETYA 1998; DES 2000; DETYA 2000; DfEE 1999; Lindenskov & Wedege 2001). Developing countries are also experiencing a push for adult numeracy, along with adult literacy (Clements 2001) — although the latter is usually regarded as of prime importance. (See also FitzSimons, Coben, & O'Donoghue 2003, for an overview of lifelong learning and mathematics education.)

Historically, archaeological studies of Egypt and Mesopotamia reveal a close, symbiotic relationship between mathematics and writing, based on the need to measure, divide and distribute the material wealth of societies. Without writing, the limitations of human memory limited the degree of numerical sophistication. Conversely, material needs, particularly the need for record keeping, were central to the development of writing (Ritter 1989). Ritter observes that no word for "mathematician" existed in these ancient languages. Rather, there were scribes who could become mathematics teachers or work as accountants — to calculate work, rations, land and grain. Accordingly, it may be suggested that the earliest mathematics was actually a form of numeracy, predating literacy. Only when it became a codified set of abstractions could it be called a discipline. From a different perspective, Bishop (1988) describes mathematics as a pan-cultural phenomenon, founded on what he claims to be six 'universal' activities: counting, locating, measuring, designing, playing, and explaining. In other words, all people — young or old, schooled or unschooled — in all cultures normally perform such mathematical activities during the course of their lives. As a discipline, mathematics is distinguished from science in the possibilities it offers for acting upon ideas and concepts from other fields. In the 20th century, Kahane (1998) observed that, unlike other sciences, mathematics is not defined by its subjects in nature or society. ". . . mathematics acts on notions coming from different fields, generalizes, simplifies, purifies, makes a theory out of them, with mathematical definitions and deductions. Then and only then are these notions available to the unexpected" (p.83).

Numeracy in the Australian Context

In Australia, the idea of employment-related competencies was developed by the Finn committee in 1991 as part of a series of reports focusing on young people and their employment. This idea was passed onto the Mayer committee to develop a set of generic *Key Competencies* (Mayer 1992), described as being essential for effective participation work and in other social settings. They are as follows: (a) collecting, analysing and organising information, (b) communicating ideas and information, (c) planning and organising activities, (d) working with others and in teams, (e) using mathematical ideas and techniques, (f) solving problems, and (g) using technology. Since 1992 the term 'using mathematical ideas and techniques' has tended to be replaced in common parlance by 'numeracy skills'. More theoretical distinctions will be made and their consequences discussed below. According to Virgona et al. (2003), although numeracy as a generic skill is regarded abstractly by policy makers, for teachers and students it is only when it is taken in context that it becomes meaningful.

The Adult Literacy and Numeracy Competency Scales.

Griffin and Forwood (1991) conducted a literature review to underpin the development of their adult literacy and numeracy competency scales. Although accepting the argument of Chapman and Lee (1990, cited in Griffin & Forwood 1991) that the competences of literacy encompass numeracy as the reader engages in the content area of mathematics, they claimed that "a distinction needs to be made between the ability to comprehend text containing quantitative information and the ability to perform the mathematical operations which may be needed to solve specific numeracy problems" (p.17). They do not support the notion that numeracy is only about dealing with the verbal language of mathematics given the tendency of quantitative information to be transmitted in a variety of means, including the symbolic registers of mathematics. They noted the work of Halliday (1978, cited in Griffin & Forwood 1991) with regard to the register of mathematics, which "demonstrated that the language of mathematics draws upon a range of language types, redefines existing words, coins new ones and develops a degree of what he called nominalisation[1]" (p.17). Because of the varying degrees of technicality and abstraction according to context, Griffin and Forwood argue that an increasing control of mathematical skills will be necessary to enhance the individual's ability to access and choose from a range of registers and hence understand what the text is signalling, to define the purpose of the information, and to make the necessary decisions. Thus, they argue that there are two major components of numeracy: *mathematical literacy*

[1] According to Fairclough (2000) this term is the representation of a process as a noun, obfuscating agency and responsibility.

— understanding the concepts and the register of mathematics, and *mathematical competency* — operationalising the skills, applying the processes of mathematics.

Griffin and Forwood (1991) arrived at the following definition of numeracy:

> Numeracy has been defined for this study as the ability to process quantitative information and to apply basic arithmetic and other mathematical operations. An increasing ability to process, apply and reflect upon quantitative information in a range of contexts represents the development of numeracy.
>
> Increasing proficiency can be described by discrete competencies, which engage a progressively increasing control of mathematics operations and understanding both within and outside the context in which the operation was initially learned. (p.19)

The Numeracy Scales are outlined in a series of appendices. *Appendix D, Numeracy: Basic Operations* (10 levels), goes from counting, whole numbers, the four operations (including long division), fractions, decimals, percentage, to calculation of circumference and creation of useful formulae and understanding of indices. *Appendix E, Numeracy: Measurement* (9 levels), goes from measurement units and geometrical names up to area and volume calculation and conversions. *Appendix F, Numeracy: Quantitative Information Processing* (9 levels), outlines numeracy activities under the following headings:

A Time and Basic Numerical Value

B Order, Changes and Scales of Measure

C Common Relationships and Operations

D Planning and Organising with Quantitative Information

E Interpretation and Analysis

F Evaluate and Apply Numerical Information

G Extract and Manipulate Numerical Information

H Insight, Inference and Critical Skills

I Numerical Reasoning and Inquiry

The last takes the most critical approach to numeracy and recognises the value of artefacts. Although eschewing the notion that assessment scales should form a de facto curriculum, Griffin and Forwood's (1991) scales envisage the concerted development of mathematical skills, albeit in the context of everyday and workplace use, including the development of a critical perspective in the learner.

The National Reporting System [NRS] (Coates et al., 1995) was developed "as a mechanism for reporting the outcomes of adult English language, literacy and numeracy provision in the vocational education and training system, in labour market programs and in the adult, comyy education sector" (p.1). There are five levels of competence and for each level of numeracy there are four features: meaning-making strategies, problem solving strategies, mathematical knowledge, and mathematical representation — each with multiple performance strategies listed as behavioural exemplars typical for that level. NB definition *

In recent reviews of numeracy published by the NCVER there are no clear definitions of what is meant by numeracy, except as a subset of literacy skills: "literacy includes the recognition of numbers and basic mathematical signs and symbols within text" (Falk & Millar 2001, p.9). Watson, Nicholson, and Sharplin (2001) declare that attempts at a single definition are relatively futile, and ANTA is quoted to define numeracy merely as calculations needed in the workplace (Sanguinetti & Hartley 2000). In the Kearns review, which stresses an increasing demand for generic skills, the word *numeracy* occurs several times, but the concept is neither defined nor problematised. Numeracy, in relation to basic skills, is assumed to be an important pre-requisite for employability.

The fact that these ANTA review documents are premised upon literacy and numeracy being taught together and integrated into workplace training results in them being treated as a single entity throughout these reviews of research, and in a related guide for practitioners (ANTA 2000). Even the Falk and Millar (2000) review treats numeracy as an appendix to literacy. Sanguinetti and Hartley (2000) have identified a range of problems which arise from this situation, including:

- Implicit numeracy competencies in industry standards require a high degree of analytical sophistication and educational expertise … not all Enterprise-Based trainers nor workplace trainers have such expertise. Often buried in training packages, literacy and numeracy competencies need to be made more explicit. (p.33).

- The assessment-driven model minimises need for teaching or support; there are limited opportunities for development of underpinning skills. More holistic and structured approaches are required. (p.34)

However, the Dawe (2002) report on generic skills in training packages acknowledges that while the integration of generic and technical skill development provides for easier transfer of generic skills, there is still a place for the separate and prior teaching of specific language, mathematics or learning skills — especially in engineering programmes. Interestingly, this report also revealed that very little if any mathematics or numeracy knowledge appeared to be required in many of the competencies in ten training packages which were examined for the Mayer key competencies. For example, in the Agriculture Training Package two of the six mandatory units of competency included 'Use hazardous substances safely' and 'Plan daily work routine'. Yet the report noted, "in general, these units did not include using technology or mathematical ideas and techniques" (p.40). This suggests that perhaps there was a lack of recognition of the actual numerical practices which occur in workplaces but which are embedded in other Key Competencies.

As will be discussed below, the international mathematics education research community distinguishes between mathematics and numeracy, yet maintains that numeracy must be underpinned by mathematical knowledge of an appropriate kind.

Numeracy is taken to be much broader than facility with numbers or basic arithmetic, and includes spatial and quantitative (statistical) literacy. Throughout the Kearns (2000) report on generic competencies, there was a stress on problem solving, systems thinking, and analytic skills (generic numeracies, according to Buckingham 1997). These are essentially mathematical cognitive skills and, I assert, cannot develop into agentic behaviours unless there is strong underpinning disciplinary knowledge to support them (FitzSimons 2002).

International Perspectives on Numeracy

The 1982 Cockcroft Report was commissioned to enquire into the teaching of mathematics as it related to further and higher education, employment and adult life generally. Although there were some people who could cope confidently and competently with any situation requiring the use of mathematics in everyday life, there were many others for whom the reverse was true. The report observed (Cockcroft 1982):

> The extent to which the need to undertake even an apparently simple and straightforward piece of mathematics could induce feelings of anxiety, helplessness, fear and even guilt in some of those interviewed was, perhaps, the most striking feature of the study. (p.7)

There were no connections found between levels of qualifications and the extent of mathematics usage; between mathematical competence and occupational group; or of self-estimates of mathematical competence and level of usage. People with high academic qualifications experienced particular feelings of guilt about their lack of confident understanding of mathematics as there was a general imputation of superior mathematical ability. Others felt guilty that they had not used the ‘proper’ standard classroom algorithms. For many, if their single method for solving a problem failed, they lacked the ability and confidence to use an alternative approach — even an awareness of the possibility. Some felt that there always had to be an exact answer, and were unable to approximate or round off results. Others expressed long buried anxieties about speed and accuracy, as well as the requirement to show a neatly written solution, often to demonstrate a method that they had not personally used.

From the USA, Steen (2001) makes a clear distinction between mathematics and numeracy, arguing that students need both. Whereas mathematics requires a distancing from context, numeracy (or quantitative literacy, in his words) is anchored in real data that reflect engagement with life’s diverse contexts and situations. It offers contingent solutions to problems about real situations (Steen 2001).

From the Netherlands, van Groenestijn (2002) has the following working definition of numeracy:

> Numeracy encompasses the knowledge and skills required to effectively manage mathematical demands in personal, societal and work situations, in combination with the ability to accommodate and adjust flexibly to new demands in a continuously rapidly changing society that is highly dominated by quantitative information and technology. (p.37)

In the UK, Evans (2001) proposes a provisional working definition of numeracy:

> Numeracy is the ability to process, interpret and communicate numerical, quantitative, spatial, statistical, even mathematical information, in ways that are appropriate for a variety of contexts, and that will enable a typical member of the culture or subculture to participate effectively in activities that they value. (p.236)

This may be compared with the definition by Coben (2003), also from the UK, citing her own earlier work:

> To be numerate means to be competent, confident, and comfortable with one's judgements on *whether* to use mathematics in a particular situation and if so, *what* mathematics to use, *how* to do it, what *degree of accuracy* is appropriate, and what the answer means in relation to the context. (Coben 2000: 35; emphasis in the original) (p.10)

While Evans's definition specifies a breadth of cognitive outcomes (what numerical practices might be possible), Coben's places more emphasis on how they might be achieved, raising the question as to whether mathematics is necessary at all. (See also Keitel, Kotzmann, & Skovsmose 1993, pp.272-273.) In relation to the task of teaching and learning numeracy on the job in chemical spraying and handling, Coben's (2000/2003) definition has most resonance.

Coben (2002, p.28) also discusses the concepts of use and exchange value in the discursive domains of adult numeracy teaching, emphasising distinctions between adult numeracy as skill, practice, learning, and education. She summarises the major differences in perspectives in the figure below, labelling the domains as 'One' and 'Two', and notes that "… adults may engage simultaneously in more than one discursive domain of numerate practice and they may be aware of doing mathematics (or practising numeracy) only in Domain One" (p.29).

	Domain One use value low; exchange value high	**Domain Two** use value high; exchange value low
Why?	To gain access to institutions of modernity; based on the belief that to be numerate is beneficial both to the individual and to society	To do something; to understand something (not necessarily something mathematical *per se*)
What?	Through a formalised standardised certificated curriculum, positioned as 'basic skills'	Through informal, non-standard mathematics practices which may be (dis)regarded as 'just common sense' by all concerned; invisible

		mathematics
How?	Through teaching; learning materials may be technologised, unitised, commodified	Through social activity or alone 'in your head'
Who?	Learners: those deemed to be deficient in mathematics ('innumerate') Teachers: professional experts (NB this is problematic in adult numeracy as the concept of numeracy is debated and the field of professional practice is poorly defined); non-professionals; volunteers	Learners: everyone, as part of processes of enculturation into 'communities of practice' 'Teachers': more experienced people, who 'know the ropes'
When?	At set times (except in Open and Distance Learning [ODL]	Anytime, incidental
Where?	In set locations, except in ODL	Anywhere, in context, in 'real life'; 'everyday life'; workplace; at home

Figure 1: Adult numeracy learning in Domains One and Two (Coben 2002, p.29)

It would appear that the discipline of mathematics as traditionally taught in schools and some post-compulsory settings aligns with Domain One and that how numeracy practices are learned aligns with Domain Two.

The Danish Preparatory Adult Education in Mathematics.

Lindenskov and Wedege (2001, p.5) proposed a two-pronged general definition of *numeracy* describing "a math-containing everyday competence that everyone, in principle, needs in any society at any given time:

- Numeracy consists of functional mathematical skills and understanding that in principle all people need to have.
- Numeracy changes in time and space along with social change and technological development."

Their analytic model has four dimensions of numeracy:

0. *Media*. More comprehensive than genre in literacy, it includes written information and communication, oral information and communication, and concrete materials, time, and processes. Literacy always concerns written texts whereas numeracy can also include non-written tasks such as working with piles of sand, areas of floors, distribution of work hours — often through oral communication.

0. *Context*. Firstly in the linguistic sense to help elucidate meaning; related to the task. Secondly, in the situational sense (e.g., historical, social, psychological relations) of something that has happened or is to be observed and considered. Examples concern work, family, education, social, and leisure time.

0. *Personal intentions.* These may be conscious and unconscious (e.g. consider the various reasons for reading a newspaper article with quantitative information).

0. *Skills and understandings*. Related to the other three dimensions, it includes the handling and sense of: quantity and numbers, dimension and form, patterns and relations, data and chance, and change.

Lindenskov and Wedege (2001) observe that:

> Whereas ethno-mathematics, folk mathematics, street mathematics etc. are analytically descriptive concepts of competence, numeracy can be both descriptive (what do we actually use?) and normative (what is desirable?), like other concrete concepts of competence (cultural competence, communication competence, social competence, etc.), which involve judgements and estimations based on values and norms. (pp.15-16)

They then propose a two-level numeracy curriculum for adults with limited previous education in mathematics. The first level, *Figures and Quantity*, intends that

> the participants clarify, improve and supplement their number sense and functional arithmetic skills for everyday practical use and personal organisation. The education is to ensure participants the possibility of developing their mathematical awareness and the ability to deal with, process, evaluate, and produce math-containing information and materials, as well as being able to communicate about these things. (p.20)

Under the heading of Content, the activities listed include: counting, measuring, locating, playing (Bishop 1988). Relevant data and media are specified, and the mathematical operations and concepts include: number work, simple fractions, percentages, and simple measurement.

The second level, *Patterns and Connections*, has as it aim:

> that participants clarify, improve and supplement their understanding of functional arithmetic and mathematics skills in practical situations and in the organisation of their everyday lives. Furthermore, they develop their functional skills at interpreting, producing and reflecting on numerical, statistical and graphic information, as well as the ability to communicate this

> knowledge. The teaching ensures the participants the possibility of further developing their mathematical awareness and of preparing for possible further education. (p.22)

The activities include all six of Bishop' (1988) 'universals' — including designing and explaining in addition to those listed above. The mathematical operations and concepts include:

- Proportions and proportionality
- Simple formulas from everyday contexts
- Systems of units and scale
- Triangle, quadrangle, circle, box, cylinder
- Symmetry in patterns
- Area and volume of planes and spatial figures in everyday life
- System [sic] of co-ordinates and simple graphs
- Bar and pie charts
- Mean and dispersion
- Simple combinatorics. (p.23)

This project, as a curriculum development project rather than an assessment project, offers the possibility of participants constructing new knowledge (new for them, at least).

In Ireland, John O'Donoghue (2000) and colleagues have developed a numeracy assessment framework. In England, Diana Coben (2001 2003) teases out the tensions and contradictions in the new adult numeracy core curriculum: for example, between social inclusion and economic competitiveness, between maximising efficiency and developing creativity, between individual fulfilment and the need for constant retraining in an insecure job market. The challenge for adult educators is to work around the government's insistence upon prescriptive content set in spurious so-called adult contexts. Clearly there is a need for more research into the mathematical needs, understandings, and practices of adult learners; there is also a need for ongoing evaluation of any numeracy curriculum, wherever the location and whatever the political status.

PISA: The Programme for International Student Assessment.

This programme has been designed to assess, among other things, the mathematical literacy of 15-year old students in 32 countries around the world. According to the OECD/PISA (2002) website, the definition of mathematical literacy is as follows:

> Mathematics literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded mathematical judgements and to engage in mathematics, in ways that meet the needs of that individual's current and future life as a constructive, concerned and reflective citizen. *[Home definition]*

The website emphasises that the term *literacy* was chosen to emphasise that mathematical knowledge and skills as defined within the traditional school mathematics curriculum do not constitute its primary focus. Instead, the emphasis is on mathematical knowledge put to functional use in a multitude of different contexts and a variety of ways that call for reflection and insight. However, it recognises that "mathematical literacy cannot be reduced to, but presupposes, knowledge of mathematical terminology, facts, and procedures, as well as skills in performing certain operations, and carrying out certain methods." It is also stated that the definition should not be seen to be limited to the functional use of mathematics — aesthetic and recreational elements of mathematics are also encompassed within the definition of mathematical literacy. Finally, PISA makes no attempt to assess affect, but considers this to be implicit in numerate behaviour.

The assessment tasks include components of content (broad mathematical concepts underlying mathematical thinking), process (in increasing degrees of complexity), and situational uses. In practice, these assessment tasks have been criticised for failing to design tasks that value the knowledges of all groups in society (e.g., Jablonka 2003), even to the point of discounting practical knowledge in favour of a 'universal knower' perspective where the calculated answer takes precedence over life experience.

The Adult Literacy and Lifeskills [ALL] survey.

According to the OECD-sponsored *Adult Literacy and Lifeskills* [ALL] survey (Gal et al. 1999, p.16):

> *numerate behaviour involves*: *managing a situation or solving a problem in a real context* — such as everyday life, work, society, and further learning — *by responding* — through identifying or locating, acting upon, interpreting, and communicating — *to information about mathematical ideas* — such as quantity and number, dimension and shape, pattern and relationships, data and chance, or change — *that is represented in a range of ways* — such as objects and pictures, numbers and symbols, formulae, diagrams and maps, graphs, tables, and texts — *and requires activation of a range of enabling knowledge, behaviours, and processes* — mathematical knowledge and understanding, mathematical problem-solving skills, literacy skills, and beliefs and attitudes.

Clearly mathematical knowledges and skills, together with dispositions, are encapsulated in this definition of numerate behaviour. However, the definition appears to stop short of the possibility of new knowledge construction by the person acting in context. The working draft outlines five fundamental ideas that are thought to characterise the mathematical demands met by adults in diverse situations at the beginning of the 21st century:

0. Quantity and number

0. Dimension and shape

0. Pattern and relationships

0. Data and chance

0. Change

The ALL survey takes a broad focus, according to the Numeracy Working Group (1999) which states:

> Numerate behavior obviously includes the ability to calculate or manipulate symbols but it is far from being limited to it. In a large-scale survey context, assessment of numerate behavior can be accomplished through tasks couched in realistic non-school settings, with limited use of formal notations, and with significant presence of text-rich tasks, as well as some tasks where opinions rather than computation are called for (e.g., when interpreting statistical messages). (p.2)

The aim of this project is to enable international comparisons of numeracy levels of adults. Although it is not a curriculum project, comparative assessment surveys such as this have the potential to influence national curricula.

The National Basic Skills Survey of Adults in England 2002-3.

As an alternative to the ALL survey, due to concerns about its methodology, researchers in the UK developed *The National Basic Skills Survey of Adults in England 2002-3* (Gillespie, 2003). According to Gillespie, consideration needed to be given to: (a) assessing the full range of adult numeracy abilities, (b) acknowledging the altruistic behaviour of participants, (c) not discouraging participants through a succession of difficult questions, (d) recognising the likelihood of 'spiky' profiles according to individual strengths and weaknesses, and (e) limiting the survey to no longer than 30 minutes. Gillespie also recognises some limitations: (a) questions took the form of pre-determined multiple-choice items from the national core curriculum, and (b) there was no opportunity for discovering reasoning behind adults' answers. In addition, this core curriculum was not based on the actual performance of adults, and the levels of curriculum do not necessarily constitute a natural progression for any adult. Possible next steps include revisiting a small sample of individual responses and framing them as short-answer questions, and structured interview techniques that allow examples of adults own numeracy skills to become visible. In this way, individuals' performances could be connected with the larger survey and their contexts addressed.

Critical Perspectives on Numeracy

A fundamental consideration in the development of numeracy frameworks and surveys is whose knowledge interests are being served. Following the work of Habermas (1963/1974), it could be argued that many official reports and semi- or fully-funded numeracy programmes are serving primarily *technical* or instrumental, manipulative interests, and *practical* or hermeneutic, communicative interests. Adults are to develop the instrumental skills, confidence, and competence to function effectively in the workplace, the home, and society — as producers and consumers of commodities and/or information. The third of Habermas's interests, *emancipatory*, focuses overtly on power relations, addressing questions about knowledge production and legitimation and exploring social structures which serve to maintain and reproduce the interests of those holding power.

An example of technical and practical interests in numeracy is given in the Cockcroft Report (1982):

> We would wish the word 'numerate' to imply the possession of two attributes. The first of these is an 'at-homeness' with numbers and an ability to make use of mathematical skills which enables an individual to cope with the practical mathematical demands of his [sic] everyday life. The second is an ability to have some appreciation and understanding of information which is presented in mathematical terms, for instance in graphs, charts or tables or by reference to percentage increase or decrease. (p.11)

In the last two decades many reports have focused on adult numeracy, albeit with greater consideration of the contexts in which people act, and the processes of being able to choose and use appropriate mathematical ideas (Willis 1990). Few, however, have adopted a critical stance towards the production and legitimation of power, including the impact of technological development on our society.

The recent interest in numeracy by governments around the world has been concomitant with the upsurge of interest in educational outcomes and accountability. According to Baker (1998), the UK Adult Literacy and Basic Skills Unit [ALBSU] has formulated what he terms an *autonomous model* of numeracy as culture- and value-free; determining the basic skills (supposedly) needed by all adults. Content — activities, techniques — is the essence of this dominant, autonomous view. In standards models such as these, there is no debate about which skills are central, whose standards they are, or why they are needed (see also Coben 2001). Baker continues that the existence of multiple numeracies leads to the questioning of the validity of these standards; power relations also need to be made more explicit and open to challenge and critique.

On the other hand, Niss (1994) argues against mathematics education being limited to everyday private and social uses.

> Those not in possession of mathematical competence beyond what is used in everyday life will be excluded from influencing important processes in society that have a considerable impact on their lives as individuals and citizens. If we want to stimulate a democratic development of society — as distinct from both an authoritarian one and a populist one — the fostering of intelligent and concerned citizenship is of crucial importance. (p.376)

Accordingly, he recommends a mathematics education oriented towards the ultimate end of enabling people to take positions on and to act in relation to processes significant to society and themselves as individuals. Adults who missed such opportunities in school should not be denied them in vocational education.

Some Australian educators include numeracy as part of mathematics taught to university students in (vocational) primary education and electrical engineering courses. Yasukawa, Johnston, and Yates (1995) assert that numeracy is a

> critical awareness [which] enables us build bridges between mathematics and the real world, with all its diversity. Being numerate involves not only having this critical awareness, but also involves the responsibility of reflecting that critical awareness in one's social practice. Thus, being numerate means being able to situate, interpret, critique, use, and perhaps even create mathematics in context, taking into account all the mathematical as well as social and human complexities which come with that process. (p.816)

They continue that in order to engender numeracy it needs to be socially situated, incorporating personal reflection and social negotiation; it needs to "excavate, uncovering assumptions and value systems, and whose interests are being served by any given representation of reality" (p.819). Numeracy is necessarily subjective, according to Johnson and Yasukawa. In addition, the wider institutional context will affect the extent to which teachers' and learners' ideologies can be enacted.

Klein (2000) considers numeracy not as a thing to be possessed, but as a capacity for action. Thus, in relation to numeracy, democratic power depends upon access to mathematical knowledge — information selectively derived from a range of possibilities and which is capable of being interpreted and understood — access to which is also unequally distributed. Klein argues that "numerate behaviour reflects a certain agency with mathematics and comprises intellectual and social aspects of knowing mathematics" (Klein 2000, p.76).

Reflections on Adult Numeracy

The last two decades have seen the burgeoning of new perspectives on the use of mathematical knowledge in practical situations of life and work which do not necessarily reflect the practices of the traditional mathematics classroom. Indeed, misconceived attempts to try to recall classroom practices (albeit imperfectly) often result in gross errors, and the suppression of common sense — also learned over a lifetime in school mathematics classrooms — does nothing to alleviate the situation. Yet, on the other hand, all recent surveys stress the importance of a foundation of mathematical knowledge; numeracy also goes well beyond being a sub-branch of literacy, important as it is in this field. However, no model of numeracy can be culture- or value-free. In the Australian context of training packages, the underlying assumption is that the selection of numeracy (and literacy) skills must be relevant to the particular industry. However, it appears that important connections between numeracy and occupational health and safety, for example, are being overlooked or treated as peripheral. As will be discussed further below, there is a tendency for industry personnel to see numeracy (or mathematics) as a set of isolated skills, to be transferred unproblematically to any situation.

Having reviewed a wide variety of approaches to numeracy, the definition of numerate behaviour by Coben (2003) noted above and reiterated here seems the most appropriate in the context of teaching and learning numeracy on the job:

> To be numerate means to be competent, confident, and comfortable with one's judgements on *whether* to use mathematics in a particular situation and if so, *what* mathematics to use, *how* to do it, what *degree of accuracy* is appropriate, and what the answer means in relation to the context. (p.10)

Theoretical Distinctions Between Mathematics and Numeracy

In the above sections I have reviewed various approaches to adult numeracy, each of which somehow link numeracy with mathematics. My personal interpretation is that the two may be distinguished with reference to Bernstein's (2000) concept of vertical discourse and horizontal discourse, linked with his performance and competence models of education.

Bernstein (2000) distinguishes between two fundamental forms of discourse. In the educational field they are known as: school(ed) vs everyday common-sense knowledge, or 'official' vs 'local' knowledge. Common sense knowledge is likely to be: "oral, local, context dependent and specific, tacit, multi-layered, and contradictory across but not within contexts" (p.157). Mathematics is an example of a *vertical discourse* on account of its coherent, explicit, and systematically principled structure. The discourse of traditional school mathematics recontextualises a selection from the abstract discipline of mathematics and is mostly only about mathematics, with very few external references. By contrast, the knowledges of *horizontal discourses* are "embedded in on-going practices, usually with strong affective loading, and directed towards specific, immediate goals, highly relevant to the acquirer in the context of his/her life" (p.159). The construct of *numeracy* is an example of a horizontal discourse.

According to Bernstein (2000), the pedagogy of horizontal discourses is usually carried out face-to-face. It may be transmitted by modelling, by showing, or by explicit means. If necessary, the pedagogy is repeated until the particular competence is acquired. Bernstein continues that: "From the point of view of any one individual … there is not necessarily one and only one correct strategy relevant to a particular context" (p.160). He concludes that horizontal discourse "facilitates the development of a repertoire of strategies … activated in contexts whose reading is unproblematic" (p.160).

Whereas in mathematics there is a well-known hierarchy between so-called common sense and so-called uncommon sense, with numeracy common sense is of the essence. High level abstractions alone are insufficient and may even prove counter-productive in the workplace. Numeracy cannot be said to have a specialised language, except at the most local level of use in context. For example, the use of the term "thou" [i.e., thousandths] is widely used in the building and automotive industries, but may not have meaning elsewhere. Numeracy is not necessarily explicit or precise, and its capacity for generating formal models may be limited to the context at hand rather than generalisable. Observations of workplace numeracy practices resonate strongly with Bernstein's concept of horizontal discourse and its associated pedagogy.

Performance and competence models of education.

Bernstein (2000) addresses the concept of *competence* as part of his theorisation on pedagogising knowledge. In his analysis competences are intrinsically creative, and tacitly acquired in informal interactions. Their acquisition, but not their forms of realisation, in practical accomplishments are beyond the reach of power relations. In this sense their constitution may be regarded as social, but independent of any particular culture.

By producing two contrasting general models of practice and context, *competence* and *performance* models, Bernstein (2000) shows how recontextualised 'competence' constructs a specific pedagogic practice. The performance model "places emphasis upon a specific output of the acquirer, upon a particular text the acquirer is expected to construct, and upon the specialized skills necessary to the production of this specific output, text or product" (p.44). In his broad differentiation between models, Bernstein refers to six features of recontextualised knowledge which both models share: (a) categories of discourse, space, and time; (b) pedagogic orientation and evaluation; (c) pedagogic control; (d) pedagogic text; (e) pedagogic autonomy; and (f) pedagogic economy.

In the first four of the six identified features, the performance model is characterised by explicitness and control, specifying content, grading and rules, highlighting absences or deviance, distributing blame. On the other hand, these four are weakly classified under the competence model. The fifth feature, pedagogic autonomy, is claimed by Bernstein to be greater under the competence model. While this applies to the horizontal discourse of numeracy, under the strict accountability regime imposed by CBT autonomy is reduced to the selection of available resources and methods of demonstrating competence. The final feature is also problematic in that the higher costs he attributes to the competence model are inevitably hidden — such as the costs of teachers' professional work in systems which espouse economic rationalism and where cost-cutting is a high priority. Both models appear to be converging on systems of accountability where outputs are measured and 'optimised'. This begs the question of what educational outcomes are in fact measurable.

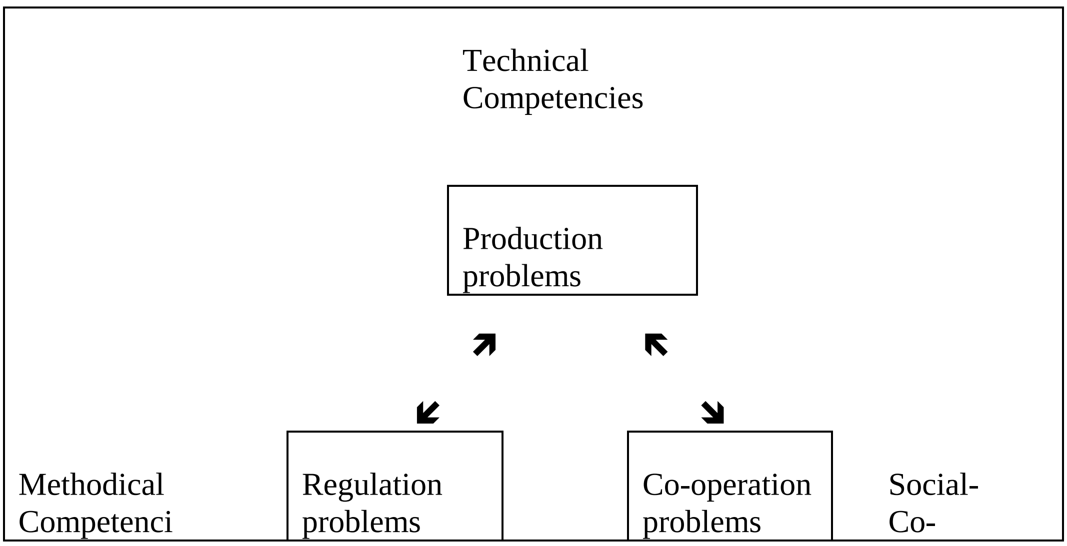
School mathematics may be contrasted with service mathematics courses at universities and vocational institutes, such as mathematics for engineering, the sciences, business, or calculations for trade subjects. In the latter there is more likely to be an interface between fields of mathematical knowledge production and practice such as in the workplace. This presents the possibility — not always realised in practice — of authentic contextualisation. The clear hierarchy of knowledge within the discipline of mathematics between so-called common sense and its opposite is in direct contrast to the workplace where common sense is valued above abstract theorisations. I argue that traditional mathematics education, based upon abstract disciplinary knowledge, appears more clearly aligned with the performance model, whereas adult numeracy — whether oriented towards the workplace or social life more generally —, based upon contextualised practice, tends to be aligned with the competence model. Depending on their orientation, service mathematics courses could be more closely aligned with one or the other.

There can be no universal conception of workplace competence, particularly in times of rapid social and economic change. The concept of workplace competence is often taken for granted, but is in fact understood in complex and sometimes contested ways. Many definitions of workplace competence presuppose a functionalist, adaptation perspective, defined and evaluated in terms of successful performance of certain given or predetermined tasks. Ellström (1998) claims that this perspective

> fails to recognise the active modification and subjective redefinition of the work task that occurs continuously and with necessity during the performance of a job. . . . In fact, as argued by Norros (1991), operators in many complex production systems are in a certain sense involved in a continuous process of redesigning and improving the system. In contrast to an adaptation view, the developmental perspective strongly emphasises that people have a capacity for self-management, and that they also are allowed and expected to exercise this capacity. (p.44)

He continues that much developmental work is complex in character, with a need to move between routine and non-routine work. Cherns (1980, quoted in Engeström 1987) observes that in contemporary workplaces "treatment becomes routine, diagnosis becomes the key". These sentiments are echoed by NBEET/ESC (1996) who recognised the requirement for systems thinking as an integral part of information literacy in the recent trends towards cross-disciplinarity and teamwork.

Following the activity theoretical work of Engeström, Onstenk (2001, p.29) defines competence as "the dynamic and developing ability to adequately handle demands, expectations and problems which (can) occur in labour practices." Arguing that an elaborate insight into the field of relevant occupational problems was needed in order to determine competences needed, he developed the model below (Figure 2) locating these within the different systems which influence the work situation — production, steering and power, and sociocultural. He then identified six categories of occupational problems: technical, methodical, organisational, strategic, co-operative and socio-communicative.



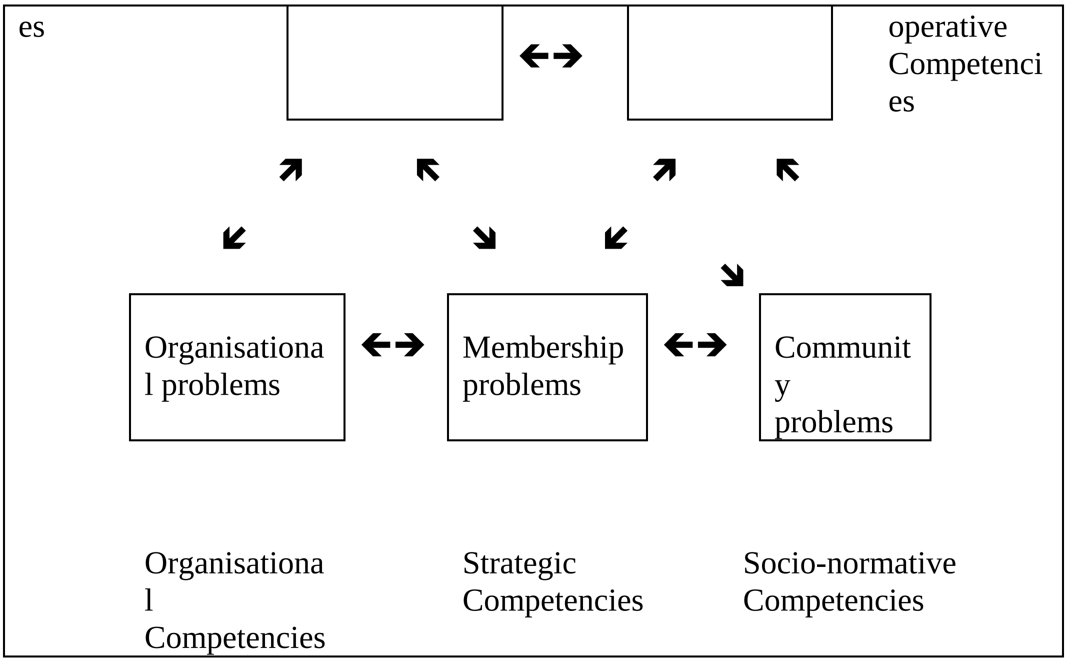


Figure 2. Core problems and broad professional competence. (Onstenk 2001, p.30)

The complexity of Onstenk's model (Figure 2) provides an indication to vocational course developers that specific disciplinary skills such as mathematics need to be located within problems that encompass the entire range of 'key' competencies and, indeed, broad occupational competence. This is in order that meaningful communication might take place, especially under flatter management structures.

Onstenk (1998) adds a further competence as a necessary element in broad professional skill: learning competence. Workers are always learning although not necessarily from formal training or in the manner (or matter) intended. This last competence, so critical to projects of lifelong learning, is more often than not taken for granted; it refers to metacognitive skills which need to be consciously developed and, I argue, requires the expertise of professional educators working in collaboration with workplace personnel.

Over the last three decades there have been reports in many countries about the mathematics (or numeracy) skills required in the workplace. Writers in previous decades have focused on mathematics in the workplace from an unproblematic 'tool box' mentality, promulgating supposed correspondences with lists of school mathematics topics. These reflect, more often than not, the industry respondee's previous school mathematics experience (Harris 1991) rather than actual mathematical practice in all its complexity.

More recent studies have researched workplace mathematics (or numeracy) — that is, how mathematical ideas and techniques are used practice, as distinct from in the school classroom. These include the following (unless otherwise stated, studies were conducted in Australia.): short-term work shadowing of workers in 30 workplaces (AAMT 1997); operators in the light-metals industry (Buckingham 1997); studies of 3 companies or organisations in each of 7 key UK sectors (Hoyles, Wolf, Molyneux-Hodson & Kent 2002); front-desk motel and airline staff (Kanes 1997a 1997b 2002); landless peasants in Brazil (MST) (Knijnik 1997 1998); supermarket shoppers in the

USA (Lave 1988); CAD/CAM technicians in Slovenia (Magajna & Monaghan 2003); carpet layers in the USA (Masingila 1993); in the UK — commercial pilots (Noss, Hoyles & Pozzi 2000), merchant bankers (Noss & Hoyles 1996a 1996b), and nurses (Noss, Pozzi & Hoyles 1999; Pozzi, Noss, & Hoyles 1998); draughtspersons in Germany (Straesser 1998); spreadsheet use by UK police (Wake, Williams, & Haighton 2000), studies of 11 UK workplaces (Wake & Williams 2001); work shadowing of semi-skilled operators in Denmark (Wedege 2000a 2000b 2002) and swimming pool construction workers (Zevenbergen 1996).

These studies show that mathematical elements in workplace settings are subsumed into routines, structured by mediating artefacts (e.g., texts, tools), and are highly context-dependent. The mathematics used is intertwined with professional expertise at all occupational levels, and judgements are based on qualitative as well as quantitative aspects. Unlike students in the majority of traditional school mathematics classrooms, workers are generally able to exercise a certain amount of control over how they address the problem solving process, albeit within the parameters of the expected outcome of the task at hand, regulatory procedures, and available artefacts. (The quality of management and effective supply and maintenance of necessary equipment are another issue.) Finally, because the focus is on task completion within certain constraints (e.g., time, money), mathematical correctness or precision can be somewhat negotiable, according to the situation at hand.

In a project designed to promote an informed view of the Key Competency, *Using Mathematical Ideas and Techniques*, the Australian Association of Mathematics Teachers [AAMT] (1997) collected about thirty reports of mathematics used in the workplace. They were based on accounts collected from mathematics teachers who shadowed workers, paid and unpaid, for about half a day. These work stories, as they were called, indicated that in fact all of the key competencies came into play in complex ways. In approaching a practical task it appears that workers bring to bear a range of skills, attitudes, and knowledges which include: (a) situational (vocational and other) knowledges and skills (in particular, mathematical); (b) meta-cognitive skills and strategies (e.g., critical thinking, planning, problem solving, and evaluating); and (c) personal skills (e.g., communication, working with others, and understanding the culture), together with certain attitudes and dispositions toward the work and the workplace. The AAMT report summarises the key elements of using mathematics for practical purposes as:

- Clarifiying the outcomes of the task and deciding on what has to be done to achieve them.
- Recognising when and where mathematics could help and then identifying and selecting the mathematical ideas and techniques to be used.
- Applying the mathematical ideas and techniques, and adapting them if necessary to fit the constraints of the situation.
- Making decisions about the level of accuracy required.
- Interpreting the outcome(s) in its context and evaluating the methods used.

(p. 59)

The report concludes that although many practical situations require an understanding of some mathematics, this would always be specific to the particular work context. However, it was unable to determine any criteria for making a difference between good and poor performance. The aim of the project was to make recommendations concerning the incorporation of the mathematics key competency into general education. However, as can be seen from the text above, there are indications of a ‘tool-box’ mentality, and certain aspects have be criticised for their tendency towards essentialism (Kanes 1997b).

My own experience (FitzSimons 2000) in the pharmaceutical manufacturing industry provides support for the notion of the often invisible complexity of the workplace. In Australia this healthcare industry is highly regulated from a legal point of view; every procedure must follow strict guidelines, set down for workers in Standard Operating Procedures (SOPs) which are constantly updated. Accountability is of the essence and is policed by regular and random audits. At each stage of the production process every item — including packaging, leaflets, raw materials, creams, and tablets — must be recorded, counted or measured, checked and re-checked by designated operators. In the case of any discrepancy, at the very least the immediate production process is halted, and in the worst case the manufacturing licence is under threat, or product recall takes place with possible loss of consumer confidence. After a discrepancy is discovered, if non-trivial, problem solving activities ensue calling upon a broad range of competencies from workers until a solution is found. Using Onstenk’s (1998, 2001) model of workplace competencies it is possible to provide an illustration of the kinds of questions which may be asked by a line manager following a (hypothetical) breakdown in production processes:

- *Technological*: Was the machine functioning correctly according to specifications? Were the raw materials as specified in amount and quality?

- *Methodical*: Were the controls set appropriately (accurately, in-time, etc.)?

- *Organisational*: Were the appropriate staff (e.g., the optimal number, sufficiently qualified) performing their allocated tasks?

- *Strategic*: How am I going to deal with this problem? How serious is it, and what are the implications of my decision?

- *Socio-normative*: Which are the most appropriate staff to help find the most efficient and productive solution? (N.B. union demarcations, availability of staff)

- *Social-Co-operative*: How will I seek help without alienating the workers concerned? Can I ‘borrow’ workers from another task? Is the delay period in waiting for the engineer, maintenance person, etc. negotiable?

Breakdowns of any kind mean time and money to an enterprise. In the allied food-processing industry the worst case scenario of product recall is becoming commonplace internationally. Working in a situation of economic contingency requires not only explicit mathematical knowledge but also a broad base of implicit knowledge together with higher order skills associated with systems thinking.

From the UK, Noss (1997) argues that sophisticated mathematical skills are required for interpretation of results as well as error detection or retrieval from catastrophic technological breakdown situations. He observed that, although in many work situations there is less reliance on traditional school mathematics skills which can be carried out more efficiently by computers, there is a greater reliance on an ability to think in a mathematical way. Noss, Hoyles, and Pozzi (1998 2000) and Pozzi, Noss and Hoyles (1998) found that there are complexities in relations between professional and mathematical knowledge, and workplace decisions are based on an interplay of these. Communication is critical, and in non-routine situations workers may draw upon underlying models which are rarely articulated, and which may indeed not have a mathematical orientation.

In Denmark, Lindenskov and Wedege (2001) propose that:

> Numeracy in the workplace can be perceived as skills and understandings charged with media, context and intention, interwoven with other competences and qualifications, interacting with the organisation of work. An instance would be the counting of items in a work situation:
>
> One does not simply count. There is a work-related aim in counting, and a certain precision is demanded. There are certain limits to the time consumption. Often one already knows the items that are to be counted. Often the shape of the items and the arrangement of the workplace will call for a special way of counting. Finally it is the organisation of work that determines who counts, controls and documents, whether it takes place individually or in co-operation, and who can suggest changes. Counting in a work context is not only counting. (p.12)

The effect of technological changes on people's lives in terms of increasing the complexity of mathematical or numeracy demands at work and beyond has been observed by many authors. For example, Lesh (2000) notes that the *USA Today* newspaper contains editorials, sports, business, entertainment, advertisements and weather sections which are filled with tables, charts, graphs and formulas "intended to describe, explain, or predict patterns or regularities associated with complex and dynamically changing systems; and the kinds of quantities that they refer to go far beyond simple counts and measures …" (p.179). He gives the example of used-car advertisements which no longer simply state the selling price, but which are instead mathematised to incorporate repayment terms and conditions. These complexities of personal, civic, and professional life are echoed by Norton Grubb (2002) who observes that mathematics in the workplace often involves a complex series of applications of relatively low-level mathematics to ill-defined problems.

Based upon studies of numeracy in workplaces of the USA, Lloyd and Mikulecky (1998a) noted that in the banking industry

> mathematics was usually called for to solve problems while gathering information from several sources, often requiring the use of technology (i.e.,

> computer or calculator), and with a good deal of speed and accuracy since customers were often waiting for answers to questions. (p.4)

Beyond banking, they observed four general types of numeracy in the workplace:

0. Calculation — where the context often determines the method used and sensible estimates are essential.

0. Measurement — requiring care and accuracy, and appreciation of tolerances.

0. Handling data — much computerised; e.g., printed graphs, charts, blueprints.

0. Problem-solving — including skills of co-operation such as explaining clearly, listening carefully, and reaching consensus.

In terms of actual worker skills, they found that:

> … only the most basic jobs are limited to the simple use of addition and subtraction … Some degree of form filling is called for in 70% of jobs and this often involves taking information from one source (e.g. a table, chart, or machine display) and performing some kind of calculation upon the information. … Much more typical of changing workplaces, however, are tasks … which call for problem-solving, setting up computations, gathering information from several sources, and estimating to check the reasonableness of answers. Researchers have consistently found the vast majority of workplace materials — manuals, memos, new product information, trouble-shooting directions — to be of high school or college level difficulty. (p.9)

Not surprisingly, there are many workers in the USA whose numeracy skills fall far below acceptable levels in any given lower-level occupational category. Even though there has been no equivalent survey to the National Adult Literacy Survey (National Center for Educational Statistics 1993, cited in Lloyd & Mikulecky 1998a), it may be surmised that this is also the case in Australia.

Another workplace numeracy study in the USA was conducted by Smith (1999). In the automobile manufacturing industry, spatial and geometric reasoning, including visualisation, and problem-solving are essential skills. He observes that:

- Mathematical work in manufacturing is tool rich. Workers who are thinking mathematically use manual and digital tools to measure, compute, represent, or program.

- Numbers and computations come from measuring physical quantities that really matter in production.

- Workers need to know the conceptual qualities of averages in times of problematic data.

- Assembly work does not require 'more mathematics' but mathematics that is used and interpreted in context.

- Some high-volume sites directly involve production workers in improving productivity and quality. When the level of workers' responsibility increases, so does the range and sophistication of the mathematics required. Similarly for workplaces that produce small numbers of precision parts and tools for high volume assemblers — machining. Almost no room for error exists in setup.

Smith also produces lists of skills of measurement and of Statistical Process Control.

Wedege (2002) postulates five working hypotheses for study of semi-skilled workers:

> (1) In every semi-skilled job, problems arise that can only be solved by quantification and use/evaluation of quantitative units.
>
> (2) Tasks and functions of semi-skilled workers require relatively simple formal skills and understanding in mathematics, but, informally, they are developed in complex working situations.
>
> (3) There are systematic differences between mathematics in the workplace and mathematics in traditional teaching.
>
> (4) While semi-skilled workers think mathematics is very important in the labour market, they do not regard mathematics as something of personal relevance to them.
>
> (5) Semi-skilled workers are not conscious of their mathematics activities in their daily work and, thus, of their 'mathematical' competence. This awareness only appears in a situation where there is a job they *cannot* manage due to their *lack* of mathematics skills. (p.25)

Steen (2001), referring to the 1991 SCANS report, notes that

> problems in which mathematical expertise may be helpful do not come with course labels attached. … Some performance expectations that require mathematical competence are identified as "basic" skills (e.g., arithmetic, estimation, graphs and charts, logical thinking, understanding chance) and others as "thinking" skills (e.g., evaluating alternatives, making decisions, solving problems, reasoning, organizing, planning). Many fall under one or more advanced "competencies" involving resources (allocating time, money, material, and human resources), information (acquiring, evaluating, organizing, maintaining, interpreting, communicating, and processing), systems (understanding, monitoring, improving, and designing) and technology (selecting, applying, maintaining, and troubleshooting).

Wake and Williams (2001) observed workers at eleven sites in the company of college students in order to discuss with workers the mathematical practices identified by the workers themselves. They found helpful the notions of conflicting Activity Systems from cultural historical activity theory and distinct Communities of Practice in analysing differences between workplace and college practices that have been shaped historically by:

- Different objectives of activities

- Different social structures, such as divisions of labour, rules of communication and exchange

- Different tools, instruments and signs. (p. 6)

They noted the role played by an individual's mathematical knowledge, beliefs, concepts and language in playing a mediating role in their activity. Mathematical practice framed within the workplace Activity System was adopted as the unit of analysis. Some of the questions asked of each case study were as follows:

> - How do 'mathematical practices' in the workplace differ from those of the College?
>
> - Can the mathematics be described using a genera; mathematical competence? — Either existing or new? If not, how is the maths best described: facts, skills, strategies … or as a 'practice'?
>
> - What are the key aspects of Activity System, work process knowledge? How are they different from College? Do these facilitate or hinder student understanding of the mathematical practices?
>
> - Are there any indications as to how the worker views mathematics? Are these views different from ours/the students?
>
> - Can we identify differences in (mathematical) understanding of workers and students/ourselves?
>
> - Are there any examples of dialogue highlighting the conceptual/linguistic/conventional 'gaps', gap-bridging and problem solving?
>
> - Are there any mechanisms (processes, conventions, divisions of labour or artefacts — see below) in the workplace activity to ensure (mathematical) competence and effectiveness? Are there mal-adaptive, or dysfunctional mechanisms?
>
> - What is the role of (new) technology?
>
> - Can we identify examples of precision an estimation in worker mathematics? — How does this compare to student mathematics?
>
> - How does the student/worker/researcher perceive the difference between work and College (mathematics and life generally)?
>
> (p.18)

They found that there were two main mechanisms that served to ensure workplace competence: the use of particular tools or artefacts to reduce the cognitive load, and the division of labour where social structures which supported individuals and teams.

Curricular and Pedagogical Issues for Schools and VET

Arguing for a 'functional mathematics' curriculum, Forman and Steen (1999) suggest that

> … functional mathematics provides much greater emphasis on "systems thinking" — on habits of mind that recognize complexities inherent in situations subject to multiple inputs and diverse constraints. Examples of complex systems abound — from managing a small business to scheduling public transportation, from planning a wedding to reforming social security. At all levels from local to national, citizens, policymakers, employees, and managers need to be able to formulate problems in terms of relevant factors and design strategies to determine the influence of those factors on system performance. Although such systems are often so complex that they obscure the underlying mathematics, the skills required to address realistic programs very often include many that are highly mathematical. (p.12)

This stress on systems thinking resonates with the NBEET/ESC (1996) Australian study. Forman and Steen provide multiple examples of mathematics in life and work, encompassing statistical quality and process control, computer-based technologies such as spreadsheets and CAD/CAM, stocking and storage problems requiring systems thinking, and cost comparison and risk evaluation.

Steen (2001) has made a series of recommendations for school mathematics of the 21st century which might well be heeded by designers of adult numeracy and vocational mathematics curricula:

- Mathematics would be presented in contexts that make sense to the learner. For example, commonly used topics such as data, graphs, and logical analysis would be stressed as much as formulas and algorithms so that students see mathematics as a tool for everyday decisions.
- Interdisciplinary applications would show the relevance of mathematics in real-world situations and students would understand how mathematics is important in other subject areas and in future careers.
- All school subjects would reinforce the role of quantitative thinking as a tool for discovering and verifying insights that are relevant to other school subjects.
- By emphasizing problem solving and reasoning skills, mathematics instruction would better prepare students to deal with unfamiliar situations.
- By learning how to ask questions and demand clarity in explanations, students would develop autonomy in reasoning.
- Mathematical and quantitative skills would be linked to literacy in ways that enhance students' abilities to communicate about technical subjects.

In terms of college mathematics, Wake and Williams (2001) identified seven general mathematics competences for vocational students:

0. Costing a project
0. Handling experimental data graphically
0. Interpreting large data sets
0. Using mathematical diagrams (including plans or scale drawings)
0. Using models of direct proportion
0. Using formulae
0. Measuring

In order to better prepare pre-vocational students as future workers, they make the following observations and suggestions:

0. Given the noticeable use of relatively 'low level' mathematics used in quite complex situations and contexts, formal education curricula should emphasise the development of these skills.
0. Given the historical development of mathematics curriculum and assessment, students are generally unaware of 'non-standard' uses of mathematics. Curriculum and assessment in formal education should encourage experiences of a diversity of conventions and methods.
0. Given that the whole process of workplace activity shapes the process and meaning of the mathematics, students in formal classrooms should be able to experience activities where the mathematics is embodied in context and to use artefacts with which they have become familiar.
0. Given that students are required to transform their existing mathematical knowledge to make sense of activities in unfamiliar workplace situations, they should be prepared for this process in formal education.
0. Given that many workers actually design spreadsheet programmes in the workplace for modelling and for the recording, processing and analysis of data, students should have experience of this in formal education.
0. Given that the mathematical activity in the workplace is often addressed and solved successfully by a range of methods, including 'non-standard' methods, students in formal education should be aware that there are many and varied ways to solve any problem.

Wake and Williams (2001) concluded that college students should be set "more challenging tasks which require them to 'inquire', to work out how others use mathematics (e.g. in work contexts) and to develop problem solving skills as well as mathematical resources and confidence" (p.41).

Quoting from a teacher trainer in vocational education, Appelrath (1985 translated by Straesser, and cited in Sträßer & Zevenbergen 1996) identifies the critical function of mathematics in vocational education:

> mathematics in vocational education serves more as background knowledge for explaining and avoiding mistakes, recognizing safety risks, judicious measurement and various forms of estimation…. Not practice at the workplace but deepening of the professional knowledge, education for a responsible use of tools and machines and the understanding of and coping with everyday mathematical problems legitimizes mathematics in vocational education. (p.660)

Distinguishing between school and the workplace, Harris (1991) notes that information embedded in context "repeatedly illustrated differences between the origin, usage and techniques of mathematics at school and at work" (p.137). She continues:

> In work however, mathematical activity arises from within practical tasks, often from the spoken instruction of a supervisor and always for an obvious purpose which has nothing to do with the numbers working out well. Thus students taught to react to isolated, abstract and write commands in the specialist language and carefully controlled figures of a school mathematics class, find themselves confronted with the urgent spoken, if not shouted, instructions in a completely different context and code. (p.138)

Sträßer and Zevenbergen (1996) suggest a practical means for vocational education mathematics teachers to prepare students for actual practice:

> By seeing the learners as a community of practice, a situated learning approach valorizes the workplace as a learning community; adopts and adapts mathematical practices which are applicable to that context; and actively engages learners as participants in a problem solving approach which allows them to solve problems in a manner which is congruent and applicable to their context. (p. 660)

Drawing on web course development literature, Kanes (2003) identifies a set of principles for the learning and teaching of numeracy, together with a variety of indicative teaching methods.

Reflections on Mathematics/Numeracy in the Workplace

Workplace studies show that mathematical elements in workplace settings are subsumed into routines, structured by mediating artefacts, and are highly context-dependent. The mathematics used is intertwined with professional expertise at all occupational levels, and judgements are based on qualitative as well as quantitative aspects. Because the focus is on task completion within certain constraints (e.g., time, money), mathematical correctness or precision can be somewhat negotiable, according to the situation at hand. Breakdowns of any kind mean time and money to an enterprise, and mathematically-oriented communications could be crucial in their prevention or recovery.

Working in a situation of economic contingency requires not only explicit mathematical knowledge but also a broad base of implicit knowledge together with higher order skills associated with systems thinking. There are complexities in relations between professional and mathematical knowledge, and workplace decisions are based on an interplay of these. Communication is critical, and in non-routine situations workers may draw upon underlying models which are rarely articulated.

Mathematical work in manufacturing is tool rich. Numbers and computations come from measuring physical quantities that really matter in production. Workers need to know the conceptual qualities such as averages in cases of problematic data. Assembly (and operative) work does not require ‘more mathematics’ but mathematics that is used and interpreted in context. In many workplaces almost no room for error exists in setup.

In summary, reports of workplace numeracy/mathematics suggest that the skills required are not necessarily to be found high up in school curricula; rather they are often regarded as ‘basic’ or high-school level, but are applied in complex ways to ill-defined and ever-evolving problems which themselves may not be inherently mathematical. I wish to argue that in order for workers to participate meaningfully in the many and varied discourses of the workplace—and in other social and civic problem solving and decision-making processes—they need a strong foundation in certain forms of mathematical thinking in order to be able to communicate across boundaries. In the last decade, this has often been described as *numeracy*, a concept which extends far beyond the popular notion of rote-learned number facts and skills, and which is rarely to be found in most senior school curricula — even for those who would gain most in their preparation for working at the lower AQTF levels (Teese 2000) or for participation in civic and social life generally. Clearly mathematical skills and knowledges developed in school and vocational education colleges play an important underpinning role in workplace numeracy practices. These would be ideally developed in classrooms and on websites influenced by sociocultural and situated learning theories, where communities of practice in problem solving situations — albeit different from actual workplace communities — are nurtured, rather than a sole reliance on individualistic, skill-based tasks.

The questions arise: How can adults employed in workplaces develop their numerical skills? What does the research tell us?

Curricular and Pedagogical Implications for the Workplace

In this section I will review a selection of the literature on workplace learning. This will be followed by a review of mathematics/numeracy learning principles relevant to the workplace.

Research on Workplace Learning

Onstenk (1999) differentiates between learning on the job and on-the-job training. The former is not structured by specific pedagogical activities but by characteristics of the work itself, affording opportunities (or not) for learning dependent on whether the work situation constitutes a learning environment. He asserts that the likelihood of learning processes occurring in a particular job situation will depend upon: (a) the available skills and learning abilities of the employee, (b) the employee’s willingness to learn, (c) the on-the-job learning opportunities, (d) the availability of on-the-job training, and (e) the relationships and mutual influences of all of these. On-the-job learning is structured only by the characteristics of the work activity itself, whereas on-the-job training is characterised by specific pedagogical structuring elements. Both the job content and the work environment can open up learning possibilities, according to Onstenk, however tensions may be experienced between work objectives and the achievement of qualifications and learning by workers. He notes that "management often still lacks an imagination for an integration of work and learning" (p.14).

Billett, Cooper, Hayes, and Parker (1997) consider the issues of workplace learning as an alternative or complement to learning in institutional settings, noting that changing nature of work is demanding the ability to go beyond the routine and predictable. According to Billett et al., the strengths of workplace participation, in offering the possibility of authentic vocational experiences, include "goal-directed activities that press workers into learning which extends and reinforces their knowledge" (p.9) as well as experiences which offer engagement in routine problem-solving. Its limitations are that:

(i) not all forms of knowledge accessed in workplaces are desirable [or appropriate];

(ii) access to authentic problem solving of an increasingly complex nature is not always available;

(iii) access to expert others is limited by availability and credibility and reluctance;

(iv) expertise is sometimes absent;

(v) knowledge is possibly hidden (opaqueness); and

(vi) [there could be an] over-reliance on instructional media. (p.48)

Regardless of how authentic institutional education tries to be it is always removed to some degree from the exigencies of the workplace. On the other hand, knowledge for workplace performance is often highly complex and takes time for robust construction to develop. As Billett et al. note, both trades and professions have mandated extensive periods of workplace experience as a requirement to develop vocational knowledge. They conclude that "it is the quality and combination of experiences and guidance that are furnished in each environment which are likely to determine the robustness of the constructed knowledge" (p.47). The question arises as to how well mathematics might be incorporated into workplace education.

Arguing against industry-wide narrow specifications of functional competence, O'Connor (1994) recommends consultation by the designers of educational programmes with those worker/learners who know most about the detailed and intimate workings and requirements of the particular context layer. Such analyses would then recognise context as a dynamic component of a larger system, as well as workers' ability to theorise, problematise, resolve, and modify work practices. The notion of workers as 'unskilled' needs to be consciously and explicitly rejected, according to O'Connor.

> Educational interventions need to start from a premise of existing skills, knowledge and ability, if they are to use those skills to assist in the development and the acquisition of new skills and knowledge. There is an increasing body of research which acknowledges and respects the fact that everyday or routine tasks or job performance reveal a deeper set of understandings of the complexity (physically or cognitively) of skills and knowledge brought to bear in the performance of work. (p.278)

He continues that ethnographic studies “emphasise that the nature of work itself is collective, and almost always requires the informal collective interaction and action among individuals” (pp.281-282). The communal model which operates has greater depth than any individual knowledge base; the group develops a communal memory of problems and solutions, and provides assistance to individuals — a valuable and relevant learning asset. Definitions of workplace numeracy (for example) based on a deficit model of the worker are inappropriate. As O’Connor remarks, often basic skills assessment exercises or techniques do not adequately assess what is actually occurring in the workplace, but instead gauge “something of an individual’s ability to perform the allotted exercise” (p.272).

Brown (1998) includes among critical learning processes the encouragement of workers to reflect upon their own learning, to see beyond the surface level, and to see their own practice as continually developing rather than “the acquisition of a fixed body of knowledge or a set of immutable competencies” (p.169). He recommends techniques to develop thinking skills, while noting the barriers imposed by the failure of other workplace personnel (in the UK, at least) to value these new techniques and problem solving skills. He also stresses the importance of learner independence, teamwork and other collaborative learning, the linking of learning and assessment practices, and the development of a substantive knowledge base. The latter is fundamental to high level expertise and goes beyond the current requirements underpinning current occupational competence in order to inform future learning.

For effective work-based learning, Brown (1998) supports Billett et al.’s (1997) claim that there is no one best context of learning, nor an optimal mix between specialist expertise and broader vocationally oriented knowledge or on-the-job versus off-the-job training. The environment needs to be challenging and varied, and a balance needs to be struck between learning for work and learning through work — especially when the work is undemanding. One positive aspect suggested by Brown is that work-based learning has the potential, especially through group projects, to address a range of issues (including quality control) not currently carried out due to limitations such as time and money). There are documented case studies of these theories in practice in the Australian automotive industry (e.g., Sefton, Waterhouse, & Deakin 1994; Sefton, Waterhouse, & Cooney 1995; Waterhouse 1996). Mathematical knowledge and skills are totally integrated into the programme of work-based learning, with successful outcomes for a range of workers including several from non-English speaking backgrounds.

According to Loo (2004), much has been written about the relationships between learning styles and learning preferences with the aim of tailoring teaching methods to the ways that students prefer to learn. In a study examining the relationships between Kolb’s (1984) four learning styles and four learning types, and twelve different learning preferences, only three significant relationships were found. Due to large individual differences in learning preferences within each style and type, and small differences in learning preference, it was suggested that, overall, there are weak linkages between learning styles and learning preferences. Loo recommended that educators use a variety of learning methods and encourage students to be receptive to different learning methods rather than try to link specific learning methods to specific learning styles. In an earlier article which supports these findings and recommendations on learning styles, Klein (2003) also contends that the claim made by multiple intelligences advocates that students' intelligences can be assessed has not been validated, and that the instructional implications offered by proponents do not follow from the theory

Transfer/Transformation of knowledges and skills.

Discussing transfer from an empirical perspective involving a study of year 10 students, Misko (1998) notes that:

> This study has shown that there is no guarantee that being able to perform a skill in one context always means being able to transfer the skill to another context. However, it seems that whether or not the skill is acquired at a proficient level in the first place has a major bearing on how well it transfers to a new context. In addition the study found that students of higher mathematics achievement level as defined by their teachers are more likely to be able to transfer the skill to a new context than are those of lower achievement. However, because substantial numbers of those of high achievement also found it difficult to transfer the skill to a new context, we can say that achievement level in mathematics is no guarantee of transfer.
>
> *Transfer promotes skill acquisition*. The major finding in this study is that transfer can generally occur if the skill has been learnt to a proficient level in the first place. That is, transfer is produced by better skill acquisition. This is a heartening finding for teacher and trainers. … The findings also give increased hope to those who believe that generic skills like using 'mathematical techniques' are not only the preserve of the intelligent, but that those of less ability can also learn skills which hopefully can endure over time and can be applied to different contexts. (p.298)

Discussing the issue of transfer from a sociocultural perspective, Billett (1998) suggests that

vocational knowledge has its genesis in different levels of social development, each with its own characteristics and potential for transfer. In current vocational curriculum frameworks, goals for vocational education often relate to the disembedded socio-cultural level of knowledge, yet there is an expectation of that knowledge being transferable across communities of practice, such as workplaces with their own sets of embedded norms and values. Yet, not only are these communities distinct, but that transfer is from one type of community of practice to another (e.g., a workplace to a particular classroom). This makes the prospect of transfer across different kinds of settings 'far' (transfer to circumstances which are novel), something which does not readily happen. (p.1)

Billett (1996) proposes that knowledge transfer is not simply based upon object similarity where, for example, the same mathematics could be taught and applied in any relevant context — as is often the underlying assumption of adult and vocational mathematics curricula. Rather, it is dependent on the way knowledge is "constructed, valued and utilised in different communities of practice" (p.21). Specifically in relation to mathematics as an abstract discipline unlike other vocational studies, Billett observes:

> The prospect for transfer is likely to be based on the ways in which mathematical activity is interpreted by individuals as being similar to another form of mathematical activity. Compounding the transfer issue is that mathematical contexts are not determined by physical factors or objectives, but how they are perceived by individuals. (p.26)

Billett then makes a series of general recommendations for educators, to aid in transfer, including: enhancing connections, assisting students to embed and disembed knowledge, encouraging reflective learning, and utilising authentic experiences.

Collins and Coleman (2002) suggest that problems with transfer may arise from the situated learning perspective which holds that training should occur in a particular context in order to be transferable — for example, through apprenticeship or practicum models. They draw attention to criticisms that situated learning fails to help learners develop critical thinking abilities and to take account of mastery. To overcome these criticisms as well as issues of disruption to work routines and productivity, including the dangers of placing inexperienced workers in unsuitable situations, they recommend the creation of training environment employing simulation techniques. Ideally, such training would "prepare students to be flexible and able to accommodate deviations from specific learning contexts in critical ways" (p.10), allowing students to be aware of "the fact that variability will be present both between the learning environment and the task environments as well as among the basic skills required for the specific task" (pp.9-10).

Drawing on activity theory (Engeström, 1987, 2001) Kanes (2002) observes that "numerical knowledge is 'generic' in so far as it is an outcome of [the] activity system of numeracy. However, it is 'site-specific' to the extent that it affords the construction of numerical tools within a site-specific activity" (p.40). Rather than numerical knowledge flowing between different activities, Kanes concludes that this projection of numerical tools into the site-specific activity leads to the *transformation* of generic numeracy and the *generation* of site-specific numeracy.

In line with Engeström's (1987, 2001) concept of expansive learning, Griffiths and Guile (2003) express the following ideas about learning associated with work

0. … context (i.e. the historical organisation of curricula and work), and therefore the access provided in different contexts to artefacts and people, influences learning.

0. … learning through work experience involves mediating the relationship between different kinds of knowledge and experience developed in school and work (i.e. theoretical and everyday).

0. …opportunities to participate in forms of social practice, for example, using context-specific language to clarify understanding and resolve problems associated with different workplace ‘communities of practice’ are central to learning through work experience.

0. … work experience should assist learners and educators to create new knowledge and new educational and workplace practices. (pp.58-59)

They also raise questions as to whether learners in work experience programmes are supported to:

0. Understand and use the potential of subjects as conceptual tools for seeing the relationship between their workplace experience and their programmes of study as part of a whole?

0. Develop an intellectual basis for criticising existing work practices and taking responsibility for working with others to conceive, and implement where possible, alternatives?

0. Develop the capability of resituating existing knowledge and skill in new contexts as well as being able to contribute to the development of new knowledge, new social practices and new intellectual debates?

0. Become confident about crossing organisational boundaries or the boundaries between different, and often distributed, communities of practice?

0. Connect their knowledge to the knowledge of other specialists, whether in educational institutions, workplaces or the wider community? (p.59)

How might these be developed in institutional learning settings as well? Griffiths and Guile (p.61) elaborate on one of the main characteristics of boundary crossing as involving a process of *horizontal development*. “Learners have to develop the capability to mediate between different forms of expertise and the demands of different contexts, rather than simply bringing their accumulated vertical knowledge and skill to bear on the new situation.” Following the work of Engeström, Griffiths and Guile distinguish between different types of boundary crossing: (a) carrying out a known activity in a new context; (b) “individuals and groups using the problems which arise while undertaking a task as the basis for developing a new pattern of activity and new knowledge, polycontextual knowledge, in a new context” (p.61).

In summary, there is no definitive statement on the optimal site of learning for work, nor on whether specialist or generalist expertise is needed. Linear, hierarchical models of curriculum, unrelated to students’ lives, are not satisfactory. The research points to integrated curricula, problem-based pedagogies, and the need for the development of skills of communication and reflection, as well as thinking and transfer skills. Clearly there is an important role for mathematics education to play as a fundamental core skill. Normatively, mathematics education for the workplace may be intended to enhance the knowing of workers both subjectively and objectively, so that as individuals they may be empowered as ‘knowledge producers’ as well as ‘knowledge consumers’ — that is, to be technologically, socially, and/or democratically numerate (in the broadest sense of the term, cf., ‘literate’).

Mathematics/Numeracy Education in the Workplace

Noss, Hoyles, and Pozzi (2000) identify an assortment of methods and algorithms used in three diverse professions (banking, commercial aviation, and nursing), including 'tricks of the trade,' which were quick and efficient at achieving particular workplace goals. However the term 'efficiency' is not taken in the same sense as 'mathematical efficiency' because a mathematical orientation is not part of the work, and the methods used are generally unavailable to scrutiny. Some of the practices match with their recognised professional textbook methods, others differ. In increasingly computerised workplaces, Noss (1997) asserts that people need to gain access to underlying models; for this they need tools (e.g., graphs, variables, parameters) and means of expression (e.g., numerical, algebraic, geometric tools). He concludes that new cultures of work are redefining the boundaries for mathematics education towards holistic approaches rather than teaching sets of isolated skills.

While on the surface it appears that fewer and fewer mathematical skills are needed in a technological society, the ability to critically evaluate their uses requires greater sophistication. Modern computer technology plays a dual role in the process of using mathematics in the workplace (Noss 1997; Straesser 1998). The use of sophisticated software hides the mathematics and speeds up its disappearance, yet "the very same technology can be used to foster understanding of the professional use of mathematics by explicitly modelling the hidden mathematical relations and offering software tools to explore and better understand the underlying mathematical models" (Straesser 1998, p.434).

Noss (2002) provides a broader perspective on the interplay between school mathematics and mathematics in the workplace, teasing out some of the apparent contradictions and paradoxes. He states these in the form of five "results". He notes that

> there is a celebrated tension between forms of discourse and cognition that are delicately tuned to cultural practices, and those that are focused explicitly on mathematics *per se*, recognisable by its symbolic forms and epistemological structures. This tension parallels (and is perhaps derived from) the epistemological duality of mathematical thought as both tool and object, simultaneously as a component of pragmatic activity and theoretical endeavour. (p.47)
>
> [There is a tension between ways of thinking and acting in relation to the cultural practices of the workplace and the discipline of mathematics in both the symbols used and the ways of knowing. This may derive from the fact that mathematics is simultaneously used as a tool as well as an object of study.]
>
> Result 1: There is an epistemological fragmentation of the knowledge structure of the workplace that shapes, and is shaped by, the discourse of the working practice. Strategies are finely tuned to the pragmatic demands of work activities, with little tendency to strive for a theoretical orientation involving generality or appreciation of unifying models. (p.52)

[In the workplace strategies are finely tuned to the pragmatic demands of work activities rather than adopting a theoretical mathematical orientation.]

Result 2: Tools and artefacts shape activities and thought in ways that only become visible at times of breakdowns to routine. In disruptions to routine, individuals need to develop a broader interpretative view of the model that underpins their routine practice. (p.53)

[Tools and artefacts shape activities and thought in ways that only become visible at times of breakdowns to routine. In disruptions to routine, individuals need to develop a broader understanding of the model that underpins their routine practice.]

Result 3: Knowledge is mutually constituted by a coordination produced in activity of mathematical knowledge and situational noise to form situated abstractions. (p.54)

[delete]

Noss (2002, p.55) asks: "… how does the formally-learned knowledge … become transformed both cognitively and culturally, into something new and more functional within professional practice and what connection, if any, is maintained between them? He then answers:

> … the key issue concerns the transformation of knowledge, the creation of new epistemologies as a characteristic part of professional expertise. Here, at least, is the explanation of the apparent invisibility of mathematical activity. Here too is a broader, more culturally oriented perspective on the hitherto individualistic notion of situated abstraction that recognises the individual's embedding in an ambient social and cultural space. (p.55)
>
> [… the key issue concerns the transformation of knowledge and the creation of new ways of knowing as part of professional expertise. This explains the apparent invisibility of mathematics and also offers a broader perspective incorporating social and cultural issues, rather than just individualistic notions.]

Result 4: As mathematical knowledge is embedded in new settings and activities, it undergoes an epistemological and cognitive transformation. What is consciously thought of as mathematics by practitioners appears to be only the visible component of a larger, transformed body of mathematics in use that takes the form of situated abstractions. (p.55)

[As mathematical knowledge is embedded in new settings and activities, it is transformed. What is consciously thought of as mathematics by practitioners appears to be only the visible component of a larger, transformed body of mathematics in use.

He notes that, in the study of nursing, that "… when the texture of nursing practice became unavailable for any reason, the mutually constitutive elements of professional and mathematical knowledge became disconnected and the situatedness of their conceptualisation was apparent" (p.56).

> [delete]
>
> Result 5 (conjecture): The noise of a situation forms a core part of a situated abstraction. When it can be called upon in a new situation (and only then?) the mathematical knowledge can be 'transferred'. (p.56)
>
> [delete]

Noss (2002) believes that in the knowledge economy of the 21st century the competence in constructing, interpreting and critiquing models has become a core part of social and professional life. His (and his colleagues') research has shown that:

> a person's mathematical knowledge is not invariant across time and space; it is transformed into different guises, different epistemologies, more or less visible as mathematics. This transformation seems much more powerful than the traditional notion of "application" or "use" that is often employed as a metaphor to describe the process. (p.59)

Linking cognitive and cultural perspectives, he concludes that

> abstractions constructed within concrete situations may compensate for their lack of universality by their gain in expressiveness. When general relationships can be expressed, they can be explored and become familiar. In the process, the links with knowledge of lived-in cultures can be maintained, rather than severed in the quest for ultimate pinnacles of abstraction. (p.59)

Noss also stresses the importance of mathematical models in relation to the increased number of people who need to understand the system they are using (cf., Engeström 1987). He concludes that "the analysis of mathematics in work concerns the transformation of knowledge as it is recontextualized across settings" (p.59). Of course, this requires the mathematical knowledge or the opportunity for learning it to be present. It cannot be assumed that any worker, even with Year 12 mathematics has developed useable mathematics in school; rather, as suggested by Kanes (2003), the focus may have been on constructible mathematics. However, my own experience of 20 years' TAFE teaching suggests that, for many, the skills may have been learned in isolation, making transfer problematic, with the end result that common sense in relation to realistic answers, or even the production of reasonable estimates, is frequently lacking.

In their studies of three companies or organisations in each of seven key UK sectors Hoyles, Wolf, Molyneux-Hodson and Kent (2002) conclude:

> A key finding of this study is that 'mathematical literacy' is displacing basic numeracy as the minimum mathematical competency required in a large and growing number of jobs. Mathematical literacy is the term we have used to describe the application of a range of mathematical concepts integrated with a detailed understanding of the particular workplace context. There is a need to distinguish between numeracy, mathematics skill and mathematical literacy. (p.3)

Some aspects that the study highlights as being of significance in mathematical literacy (in relation to operative work such as chemical spraying and handling) include:

- Calculating and estimating (quickly and mentally)
- Proportional reasoning
- Calculating and understanding percentages correctly
- Recognising anomalous effects and erroneous answers when monitoring systems
- An ability to perform paper and pencil calculations and mental calculations as well as calculating correctly with a calculator
- Communicating mathematics to other users and interpreting the mathematics of other users
- An ability to cope with the unexpected. (p.5)

To develop reflective thinking Keitel, Kotzmann, and Skovsmose (1993) propose starting from a meaningful problem context, where "mathematical concepts, theoretical frameworks and modelling activities should be used to become able to understand the problem, formulate alternative solutions and negotiate with others about their acceptability" (p.275). This starting point accords well with Onstenk's (2001) problem-based model of developing broad occupational competence. Although their work was not focused on the workplace but the school classroom, the following summary by Keitel, Kotzmann, and Skovsmose (1993) may be closely compared with findings about mathematics in, and for, the workplace. For example, it shows strong resonance with the work of Kanes (1997b, p.269), in terms of contingency, specificity, and creativity of knowledge — also with Bernstein's (2000) conception of horizontal discourse in the form of workplace numeracy.

> The knowledge produced within and for the local environment is not only a reconstruction of existing knowledge but is partly and potentially "new" knowledge. It may provide information on issues which were not available so far. This gives a special and additional value to this produced knowledge.
>
> The knowledge is specific knowledge generated in specific contexts. It is potentially valid in this context but not necessarily in other contexts. . . . [I]t could have a generalising potential which should be explicated and evaluated.
>
> The knowledge is potentially useful for a specific audience. [If access is provided to others it may increase their] ability to understand their social situations and to cope with certain demands as well.
>
> For the students the generation of locally useful knowledge implies an integration of experience-based judgement with generally available and other forms of socially valuable knowledge. (pp.275-276)

In this quotation the emphasis is on learners (or workers in our case) as knowledge producers — a far cry from the experience of most people in school and vocational mathematics classes as knowledge consumers.

According to Coben (2003), the nature of adults' contexts — their possibilities and constraints — needs to made prominent and integral to the process of teaching; adult numeracy teacher education needs to be informed by a broader concept of numeracy which connects adults' numerate practices with their thoughts and feelings about mathematics — its nature, purpose, and meaning. In FitzSimons (2001a) I draw upon the work of Giroux, in particular his work on pedagogy of representation and representational pedagogies, to work towards enabling adult and vocational education students and educators to address representational practices that have the discursive power to construct common sense and textual authority in mathematics education. Wedege (1999) explores adults' blocks and resistances to learning mathematics, arguing that adults' perceptions that they have managed their lives up until now quite adequately could cause resistance to the idea of further mathematical education. This is clearly a major issue in workplaces where occupational health and safety issues are at stake.

In FitzSimons (2001b) I outline an attempt to integrate mathematics, statistics, and technology in vocational and workplace education. Recognising that in the workplace mathematics and statistics are essential for communication and decision-making and that process workers at lower classifications of skill levels are likely to be confronted with statistical charts and warnings about non-conformity, I describe some ways in which to address the challenge of making mathematics, statistics, and technology education take on real meaning within the context of the workplace. I show how it is possible for mathematics educators to work in co-operation with industry, particularly at the local level, in a way that will encourage and support lifelong learning yet remain critical of the uses to which mathematics, statistics, and technology are put.

Van Groenestijn (2002, p.303) uses seven generally accepted andragogical starting points for adult education in general to conclude that "learning mathematics in ABE [Adult Basic Education] must focus on math-as-a-tool and hence on learning-for-doing and learning-by-doing, i.e., learning –in-action." However, she cautions against total reliance on these methods "because of the risk that such functional knowledge and skills may also yield context-bound and partial knowledge and skills."

Van Groenestijn identifies six steps in problem solving which also enable adults to analyse their own ways of learning. She distinguishes three elements:

> Through problem solving learners become aware of [the] *learning process* itself *(what)*
>
> During problem solving learners may want to emphasize the *quality* of their learning *(why)*
>
> When doing problem solving learners learn how to *organize* the learning process. *(how)* (p.304)

She argues that problem-based learning offers the possibility of creating learning situations in which mathematical concepts and actions are integrated. Adults are encouraged to create their own ways of solving problems as well as developing their own notational systems.

Finally, Lloyd and Mikulecky (1998b) adopt a functional analytical approach to numeracy education for the workplace. They recommend task analysis, including interviewing relevant personnel, observation of workers performing the relevant tasks (especially where there are difficulties), collection of artefacts, and custom-design of curriculum around those analyses. This, they claim, should ideally be followed by custom-designed evaluation—although their suggestion of pre- and post-tests is problematic—as discussed above—and likely to alienate workers, even contributing to any mathematics anxiety (see Coben 2003).

Reflections on Mathematics/Numeracy Education in the Workplace

In this section I have raised some of the tensions and contradictions when considering learning in the workplace, but also offered some possibilities for future development. Certainly, workplace numeracy education cannot be approached from a traditional 'school-mathematics' mentality — no matter how authentic the (pseudo)contextualisations are. Just as I highlighted the fundamental differences between mathematics and numeracy in the first section, I now conclude that workplace numeracy education must also require a fundamentally different curriculum and pedagogy from that of school mathematics, yet encompassing underpinning mathematical knowledges and skills in ways that enable the generation of 'new' knowledge in order to solve problems which cannot always be known in advance.

Conclusion

In this review focussed on teaching and learning numeracy on the job I have explored the contested concept of numeracy by situating it in the Australian context, and then comparing various international definitions, surveys, and curricular expectations for adult learners. Coben's (2003) definition of numerate behaviour was nominated as being the most relevant to the topic at hand. However, it was recognised that the concept can never be culture- or value-free. The first main section concluded with a theoretical discussion of the distinctions between mathematics and numeracy based upon Bernstein's (2000) discussion of vertical and horizontal discourses. Distinctions between traditional mathematics teaching and competency-based training approaches and constraints were highlighted through a brief discussion of Bernstein's performance and competence models of education.

The following section reviewed a selection of recent studies into how mathematics/numeracy is used in the workplace, and identified the kinds of skills needed in different workplaces. The conditions of knowledge production are very different in formal sites of education and workplaces, and suggestions were made as to how teachers in formal educational settings may support workplace numerical practices.

The final section briefly reviewed a selection of the literature on research into workplace learning and participation. The issue of transfer is by no means unproblematic. The differences between the workplace and the institutional classroom which take the form of different activity systems were highlighted. In the former, the object is to complete a task as efficiently and effectively as possible, assisted as appropriate by the projection of numeracy as but one tool. In the latter, the object is to generally produce more text, utilising textual and other mediating artefacts, with the intended outcome of learning more mathematics. Clearly the conditions for teaching and learning numeracy are very different in these respective sites — even when a classroom lesson is designed to simulate the workplace, it can never completely capture the exigencies of actual practice. The analysis of the data to be collected from the chemical handling and spraying research project will identify some of these exigencies.

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