Mathematics in Action

Commonalities across Differences

a Handbook for Teachers in Adult Education

Mieke van Groenestijn & Lena Lindenskov (eds)
Colophon

Title         Mathematics in Action, Communalities across Differences, a handbook for teachers in adult education
Editors       Groenestijn, M. van & Lindenskov, L. (eds)
Publisher     ALL Foundation, Netherlands
Design maps   cym, Austria
Printed by    Potzkal, Poland

Grundtvig-1 project: 116676 – CP – 1 – 2004 – 1 – DK – Grundtvig – G1
• Danish University of Education, Denmark
• Adult Learning Centre Fyn, Denmark
• The Hungarian Folk High School Society, Budapest, Hungary
• Vilnius Adult Education Centre, Lithuania
• Regional Education Centre Utrecht, ROCMN, Netherlands
• Norwegian Institute for Adult Education VOX, Oslo, Norway
• Slovenian Institute for Adult Education SIAE, Slovenia
• AGORA - Association of Participants, Barcelona, Spain

www.statsvoks.no/mia


© The editors, ALL Foundation, Netherlands
www.all-for-all@euronet.nl
## Contents

Introduction to the MiA project  
1. MiA in overview  
   Lena Lindenskov  
   7

2. Numeracy as a key competence  
   Mieke van Groenestijn  
   12

3. Theory in MiA  
   Mieke van Groenestijn  
   20

4. Field experiments  
   Denmark  
   37
   Hungary  
   41
   Netherlands  
   49
   Spain  
   69

5. Setup of MiA Teacher Workshops  
   Lithuania  
   109
   Norway  
   113
   Slovenia  
   117

6. Communalities across differences  
   Lena Lindenskov  
   135

Appendices:  

Appendix 1. Results IALS  
Appendix 2. Key competences Framework LAST  
Appendix 3. Paul Ernest’s view on transfer of knowledge  
Appendix 4. Guidelines for the six steps  
Appendix 5. Pre-Questionnaire  
Appendix 6. Post-Questionnaire  
Appendix 7. Workshop Leaders Evaluation  
147
149
150
151
158
162
164
165
167
Introduction to the MiA project

From December 2004-November 2007 Denmark, Hungary, Lithuania, the Netherlands, Norway, Slovenia and Spain have cooperated in the project Mathematics in Action (MiA). The MiA project is supported by the Grundtvig action in the Socrates program of the European Commission. The aim of the project is to support quality of learning and teaching of mathematics in adult education in the EU countries and to support participation and success rates of adult learners. The target groups are teachers in adult learning institutions and teacher trainers.

As a result of the MiA project this handbook presents examples of good practices and theoretical thoughts about doing and learning mathematics in actual real life situations.

The first chapter gives an overview. The second chapter concerns important papers from the European Commission on key competences and how they set up challenges teachers in adult education.

Chapter 3 enlightens some relevant theory for MiA on what it means to be numerate, on learning in practice and on adult education, on transfer of knowledge and on six steps in problem solving. Here you will also find the MiA research questions on adults’ learning, on teaching and on the role of adult learning centres.

Chapter 4 presents MiA in practice. It presents overviews of experiences from Denmark, the Netherlands and Spain and of MiA teachers’ view on good teaching and coaching practices. Then examples of learning experiments from Denmark, Hungary, the Netherlands and Spain are described with the themes ‘You and your body’, ‘Family budget’, ‘Sales’, ‘Health’, ‘Discount’, ‘Your favourite recipe’.

Chapter 5 conveys goals, core elements and set up of MTWs, MiA teacher workshops, based on MiA theory and MiA practice. Documents for invitation, for pre- and post questionnaires and for workshop leader questionnaire are included in the appendices and are available at the www.statsvoks.no/mia.

Chapter 5 also describes MTWs in practice. It shows experiences, comments and evaluations of MiA teacher workshops in Lithuania, Norway and Slovenia.

Finally, chapter 6 resumes some results on the MiA research questions and discusses implications for professional development for teachers in adult mathematics education.

We hope the handbook can be useful for teachers in adult education.

For information or questions after reading the handbook, please contact us.

Lena Lindenskov
DPU, Danish School of Education – Aarhus University
Tuborgvej 164
2400 Copenhagen
Project Leader MiA
<lenali@dpu.dk>
1. MiA in overview

Lena Lindenskov

Introduction
Due to technological and economical changes lifelong learning is a growing challenge. An important, but difficult and complex issue to be dealt with is mathematics in adult education. Contemporary mathematics concerning both content and organisation is a critical factor.

"Mathematics in Action", MiA, is a European project with partners from seven countries. The aim is to support teachers in adult education in order to improve the quality of learning and teaching of mathematics in adult education in the EU countries and to improve participation and success rates of adult learners, by widening learning opportunities. The MiA project
- researches mathematics education in EU partner countries, especially in out-of-school situations.
- selects and develops examples of good practises.
- provides alternative ways of learning and teaching mathematics to be used alongside usual teaching methods and to be used outside as well as inside usual classroom settings.
- pilots MiA teacher workshops, MTWs, to inform and train teachers in adult education in order to adapt MiA ideas and methods to local needs and opportunities.

The goal is to provide teachers and adult learners with models and examples for how to deal with a range of real-life situations in which mathematics including competences can be further developed such as at the workplace, at home and in societal life, and to increase adult learners’ motivation by making learning more attractive and relevant.

The handbook forms a combination of data, results and theory from several sources. Examples and theories complement each other in this handbook, as they have done through the MiA project, especially in the MiA learning and coaching experiments and in the MiA teacher workshops experiments.

Promoting numeracy for adults
The promotion of the development of literacy and numeracy for adults in European countries is required by economical and technological developments. Literacy and numeracy go hand in hand. In the MiA project numeracy has been promoted, since very little attention has been paid to numeracy until now. Underpinning MiA is the European Grundtvig 1 Action, which includes supporting relevant work place qualifications, and also facilitating the personal fulfilment of the individual person, and encouraging active participation of all citizens in public life. Numeracy often plays an underestimated role in all these broad situations and is still in a developmental stage compared to literacy. Promoting the development of numeracy for European adults is still a challenge. The first step on promoting was undertaken in the ALMAB project where four European countries started a broad investigation in their ways of teaching mathematics to adults. MiA can be seen as the follow-up of ALMAB but differs from ALMAB in the sense that MiA will now focus on an in-depth analysis of learning and teaching mathematics in adult education.
The focus in MiA
MiA focuses on a few central aspects, and investigates them in depth. The first focus is on learning and doing mathematics in real life situations where adults can experience the usefulness of mathematics. The second focus is on professional development of teachers in adult education.

MiA analyses how doing and learning mathematics in real-life situations can be used in learning and teaching mathematics in adult education settings. On the other hand MiA investigates and develops coaching methods which teachers in adult education can use to support doing and learning mathematics in real-life situations.

The handbook presents examples of good practice and theoretical thoughts about doing and learning mathematics in actual real life situations, developed through the MiA investigations. The main goal is providing exemplified alternative methods that can be used alongside traditional teaching methods. This may enhance adult learners’ motivation by making learning more attractive and relevant. Thereby MiA contributes to improvement of learning and teaching mathematics in adult education in EU countries.

From invisibility to awareness
MiA aims for awareness of the importance and relevance of numerate behaviour. MiA tries to make visible that all adults actually have informal and formal skills, knowledge and attitudes as potential part of and relevant for numerate behaviour. MiA also tries to make visible how adults actually learn in their individual lives – outside formal schooling.

Terminology
MiA does not dwell into disputes upon terminology on whether you should use terms like numeracy, adult numeracy, functional mathematics, mathematical competence, mathematical literacy, quantitative literacy, or mathemacy. Instead the word *numeracy* will be used as an overall concept. Numeracy is relevant for everybody. Numeracy is more than only doing simple basic computations. In MiA numeracy includes all mathematics that is used in real life situations. It also includes relevant problem solving skills to manage situations in which mathematical activities are embedded. Becoming numerate is a lifelong learning process that develops further beyond school.

Products developed
Products developed by the MiA participants include

- A background questionnaire for teachers to make current situations in learning and teaching mathematics in adult education visible and to help teachers reflect their ways of teaching.
- Examples of good practice based on relevant theoretical background ideas.
- Some materials for organizing MiA Teacher training Workshops (MTWs), in particular:
  - An invitation for MTWs
  - A pre-questionnaire for teachers before participation in an MTW.
  - A post-questionnaire for teachers after participation in MTW
  - A manual for organising teacher training workshops, courses and seminars.
The products can be used separately or in combination by individual teachers and teacher trainers, by groups of teachers or by institutions and organisations in order to organise and manage courses based on learning mathematics in action in formal as well as in non formal programmes for adults. The products are available on the MiA website.

**The development processes**

To set a common ground the first step in the project is an exploration of teachers’ possible views on learning and teaching mathematics in adult education. The background questionnaire is meant to accomplish this purpose. The next step is the study on learning mathematics in action which goes into depth on conceptual understanding of learning mathematics in action. Presentations from Denmark, The Netherlands and Spain about their experiences on learning in action are starting points for every partner to develop and analyse a field work experiment. The third step is doing analyses of some competencies adults need to have acquired to become functional numerate. This is combined with theoretical background studies in order to design a MiA theoretical framework. The fourth step is setting up, executing and analysing coaching experiments, which lead to further development of the MiA theoretical framework. The fifth step is developing methods, materials, and organisation models for MiA Teacher training Workshops, MTWs, for implementation and dissemination of learning mathematics in action. The MTWs are piloted by the partners. Based on evaluation of the piloting experiments methods, materials and organisation models are revised and compiled in the MTW manual for teacher trainers.

**The partners**

DPU, Danish University School of Education - Aarhus University, Copenhagen, Denmark. The DPU is committed to the pursuit of excellence in teaching and research. The University School aims to promote research and postgraduate education at the highest level in the field educational studies and strives to enhance the quality of educational research and pedagogical practice in Denmark. The University School seeks to become a leading international centre for educational research and professional development. At the Department of Curriculum Research the focus is on e.g. teacher further education and teacher in service training courses, research in mathematics in everyday life and adult basic education, and research in teachers’ and adult learners’ approaches and experiences with mathematics education.

Regional Education Centre “Midden Nederland” (ROCMN), Utrecht, Netherlands is one of the largest providers of vocational and adult education and training in the Netherlands. Vocational education is given to over 20,000 students. The very large department of adult education with over 12000 adults covers Dutch as a foreign Language, Numeracy & Literacy, general education, and integrated courses offering both language and vocational contents, reintegration courses for adults returning to the world of work, etc. The centre compiles more than 20 centres in neighbourhoods of the major cities in the Utrecht area and from community centres in smaller towns and villages. Regional Education Centre Utrecht is actively promoting international mobility and co-operation.
Adult Learning Centre Fyn – Glamsbjerg, Denmark, is a county-municipal institution for adults with 800 civil registration numbers from basic level, special education, preparatory mathematics for adults (FVU- mathematics), adult mathematics education (AVU), to Higher Preparatory Education (HF) similar to upper secondary school. The centre is involved with labour market issues by co-operation with the labour market stakeholders about education in the factories. The centre participates in different national and international networks dealing with adult mathematics.

Norwegian Institute for Adult Education, VOX, Oslo, participates in several national and international networks dealing with adult mathematics. The institute is involved in initiation, development and dissemination of innovation in adult education, and in trying out and developing mathematic material for immigrants with little school experience from their native countries and poor Norwegian language skills.

Slovenian Institute for Adult Education (SIAE), Ljubljana, deals with research, development of strategies and measures, and participates as a partner in several EU project concerning adult literacy as Phare Lien: Basic Education as a Path Back Into Society, the Leonardo da Vinci European Certificate in Basic Skills, and the Grundtvig 1: Promoting Social Inclusion Through Basic Skills Learning.

Vilnius Adult Education Centre, Lithuania, counts around 800 annual learners studying at general and secondary school level, and 81 personnel. The Centre offers flexible study programs and also organizes supplementary courses. The Centre has already made its steps in applying the modular principle as methods of teaching. Adult students can choose single subject providing knowledge of a certain level and amount. The Centre participates in international projects and in local projects.

AGORA - Association of Participants, Spain
The main aim is promoting the managing capacity and initiative of people who participate in adult education. AGORA provides support to all activities made by people who take part of adult education; and offers a wide range of cultural activities, trying to cover the lack of most of these activities among wide sectors of population, especially of the neighborhood in which AGORA is placed. Today, AGORA has 2000 participants. AGORA has since 1997 participated in seven different European projects.

The Hungarian Folk High School Society, Hungary
The aims of The Hungarian Folk High School Society are foundation of folk high schools and development of folk high schools activities; guidance on issues related to the professional and financial management of lifelong learning; assistance in designing adult learning programmes; training and further training adult learning tutors, staff members at all levels; and promoting learning partnerships, basic skills education, and active citizenship training. Has participated or currently engaged in a number of European projects.

The partners cover a broad range of quite different knowledge and experiences which put together and further commonly developed in MiA create some communal trends and perspectives in Continental European Adult Mathematics Education.
The partners cover experiences concerning

- developing new national numeracy curriculum for adults,
- developing curriculum for adult numeracy teachers,
- developing theory on adult teachers’ and adult learners’ views of everyday mathematics,
- expertise in the field of numeracy training and development of mathematics training programs, involvement in creating a more tailor made approach to learning and teaching mathematics,
- making training programs more related to the day-to-day routine in companies and businesses with more practical student assignments and mathematics as part of the on-the-job-training or work placement,
- developing a teaching method suitable for second language learners,
- utilising different organisations of learning in open learning centre, distance learning, and flexible learning,
- the teachers’ development of new learning material in co-operation with adult learners at their work places,
- courses organised at the factory level,
- making mathematics by utilising experiences from daily life,
- developing learning materials about mathematics at workplaces e.g. medicine calculations,
- investigating maths difficulties,
- cross-curricular learning e.g. language and mathematics,
- developing methods to use in training adult literacy teachers,
- monitoring and evaluation methodology in literacy provision,
- involvements in different organisation forms of learning, as learning in shifts, distance learning, modules and self-directed studies,
- developing methods to be used by teachers to adjust to and learn more about what students expect of mathematics and their needs,
- using tools to support active methods of teaching, with tasks involving real life situations, in order to show the students how mathematics can be applied in reality; and to take into account their experience,
- organisation of various mathematical parties and quizzes,
- using a broad cultural perspective on learners and adult education,
- emphasising joy and amusements also in mathematics education,
- using principles from dialogical learning based on egalitarian dialogue, emphasising solidarity and classrooms, which are democratic and provide equal possibilities for decision-making and participation for all,
- compiling results from and provide further inspiration for organisations engaged in the delivery of basic skills training for low-educated, socially disadvantaged adults,
- development of learning materials and the training of basic skills teachers.
Mathematics teachers in adult education in Europe

From results from the MiA background questionnaire for teachers and from discussions among MiA participants evolves a picture of the teachers of mathematics in adult education in Europe and of the situation for mathematics in adult education. There are big differences between European countries and also between the partner countries. Only some countries offer programmes especially for adult learners and with specialised teachers and materials prepared for adults as is in Denmark and The Netherlands. In other countries programmes for adult learners are in the process of being set up. Recently programmes in mathematics / numeracy at a basic level has been set up for Norwegian adult native speakers and for second language learners, and materials have been developed.

There are many differences in education for mathematics teachers in adult education through the partner countries. In some countries education in mathematics is required, in other countries not. In some countries knowledge about adult learning in general is required, in other countries not. The majority of teachers in adult education are part time teachers. Slovenia is one of these countries, where every teacher in adult education also teaches children or adolescents. With the term adult literacy is meant basic literacy and numeracy. Adult literacy teachers are often expected to teach both. There are no specific materials for teaching math to adults in Slovenia. Teachers adapt and develop themselves materials, but there is no training available for teachers how to develop own teaching and learning materials. The learning material that is developed or prepared by teachers themselves is various, but quality is not often guaranteed.

The background questionnaire for teachers shows that teachers in some countries use primary school material more often compared to other countries, in particular to those countries where a system for adult numeracy education was developed. Most of the teachers are interested in learning more about how adults can learn numeracy the best way.

The background questionnaire also shows that there are many more female than male teachers in adult education. The average age of teachers in participating countries is between 40 and 50 years, except in Spain where the average age is around 30 years.
2. Numeracy as a key competence

Mieke van Groenestijn

Introduction

MiA builds upon results of the International Adult literacy Survey in 1996 (IALS, 1996). The study showed that about 30% of the European adult population has numeracy skills at such a poor level that it may negatively affect their quality of life, labour market possibilities and participation in lifelong learning. (see appendix 1) The study also showed that better competence in quantitative literacy corresponds with a higher participation rate in societal situations and in adult education and training. The results of PISA 2000 and 2003 (OECD 2006) confirmed that there will still be a need for adult numeracy education in the coming years, but motivation among adults to participate in adult mathematics courses is often low.

At about the same time the need for lifelong learning became more and more clear since it appears that because of the ongoing development of science and technology the world we all live in is subject to continuous change. To be able to keep up with the ongoing changes and to be equipped for the future, people cannot longer build on only knowledge and skills acquired in former school years. Besides it appears that for people who did not succeed in school, it becomes more and more difficult to bridge the gap between their own knowledge and skills and the developments that require new knowledge and skills.

In the following sections attention has been paid to the developments of lifelong learning and key competencies as presented by the European Commission in 1996, the White Paper, and by the European Council, the DeSeCo program (OECD, 2005). The text is derived from the official reports since this is important background information for teachers in European adult education.

The White Paper

The European Commission published in 1996 the White Paper, titled Teaching and Learning, Towards the Learning Society. The commission describes the need for lifelong learning for European citizens to be able to participate in the knowledge-based society. There are three factors of upheaval: (pg. 5 and 6)

- The impact of the information society: the main effects of this is to transform the nature of work and the organization of production. Routine and repetitive tasks which used to be the daily lot of most workers are tending to disappear as more autonomous, more varied activities take their place. The result is a different sort of relationship with the company. The individual worker has become more vulnerable to changes in the pattern of work because he has become a mere individual in a complex network.

- The impact of internationalization affects the situation as regards job creation. Internationalization is bringing down the borders between the labour markets, thus making a global employment closer than is generally thought.
- *The impact of the scientific and technological world:* the growth in the scientific knowledge, its application to products methods and the increasingly sophisticated products give rise to a paradox. Despite its generally beneficial effect, scientific and technical progress engenders a feeling of unease and even irrational misgivings in society.

The European Commission indicates the knowledge required to become employable in today’s world have been seen as *an acquired body of fundamental and technical knowledge, allied to social skills.* (pg. 30) Basic knowledge is the foundation on which individual employability is built. This is the domain of formal education and training systems. A good balance has to be found in basic education between acquiring knowledge and methodological skills which enable a person to learn alone. It is these which today need to be developed.

In the eighties and nineties European countries opted to re-center basic teaching on the three ‘R’s – reading, writing and arithmetic– (as is called in English) in order to prevent school failure which plays a major role in social exclusion. However, to be equipped for the future people need also technological knowledge and social aptitudes. Nowadays the emphasis is on the development of key competences, which requires rethinking the content and subjects of the three ‘R’s, in particular concerning numeracy and mathematical literacy. In addition learning languages is strongly advised due to the globalization. This leads us to the DeSeCo program.

**Definition and Selection of Competencies (DeSeCo)**

In March 2000, the Lisbon European Council set a new strategic goal for the European Union: to become ‘the most competitive and dynamic knowledge-based economy in the world, capable of sustainable economic growth with more and better jobs and greater social cohesion’. To achieve this, Europe's education and training systems need to adapt to the demands of the knowledge society and to the need for an improved level and quality of employment. One of the main components of this approach is the promotion of new basic skills: more concretely, the Lisbon European Council called upon the Member States, the Council and the Commission to establish a European framework defining ‘the new basic skills’ to be provided through lifelong learning. This framework should cover ICT, technological culture, foreign languages, entrepreneurship and social skills.

A year later, in March 2001, the Stockholm European Council adopted the report *The concrete future objectives of education and training systems*. This document identifies three strategic objectives (quality, access and openness of the education and training systems), broken down into 13 associated objectives. The Barcelona European Council (February 2002) then adopted a detailed work programme for achieving these common goals and objectives by 2010. The detailed work programme extended the list of basic skills as follows: literacy and numeracy (foundation skills), basic competences in mathematics, science and technology, ICT and use of technology, learning to learn, social skills, entrepreneurship and general culture.

The Barcelona Council conclusions also stressed the need for action to improve the mastery of basic skills. In particular, it called for attention to digital literacy and foreign languages.

---

2 Council document 5980/01 of 14/02/2001.
3 Detailed Work Programme on the follow-up of the objectives of education and training systems in Europe (2002/C 142/01)
Moreover, it was considered essential to promote the European dimension in education and to integrate it into pupils’ basic skills by 2004. Following the adoption of the detailed work programme, the Commission has established expert groups to work on one or more of the thirteen objective areas. These groups consist of experts from Member States, EFTA/EEA countries, associated countries and European-level associations. The working group on key competences started its work in 2001\(^4\).

The main objectives of the working group are to identify and define what the new skills are and how these skills could be better integrated into curricula, maintained and learned through life. There is a particular focus on less advantaged groups, those with special needs, school dropouts and adult learners.

**Principles underlying the definition of the framework for key competences**

The following text is derived from the DeSeCo report (pp 3-4) (European Commission 2004)

1. The framework is the first European-level attempt to provide a comprehensive and well-balanced list of the key competences that are needed for personal fulfillment, social inclusion and employment in a knowledge society. It aims to serve as a “reference tool” for policy-makers and for those responsible for creating learning opportunities for people at all stages of lifelong learning, allowing them to adapt the framework as appropriate to learners’ needs and contexts.

2. The terms ‘competence’ and ‘key competence’ are preferred to ‘basic skills’, which was considered too restrictive as it was generally taken to refer to basic literacy and numeracy and to what are known variously as ‘survival’ or ‘life’ skills. ‘Competence’ is considered to refer to a combination of skills, knowledge, aptitudes and attitudes, and to include the disposition to learn in addition to know-how. A ‘key competence’ is one crucial for three aspects of life:
   a. personal fulfillment and development throughout life (cultural capital): key competences must enable people to pursue individual objectives in life, driven by personal interests, aspirations and the desire to continue learning throughout life;
   b. active citizenship and inclusion (social capital): key competences should allow everybody to participate as an active citizen in society;
   c. employability (human capital): the capacity of each and every person to obtain a decent job in the labour market.

3. Given the chosen approach, namely defining the key competences in broader terms, it is neither possible nor relevant, in most of the competence domains, to distinguish between the very ‘basic levels’ of mastery of a competence from more advanced levels of mastery. The term ‘basic’ refers to something that depends on the requirements of the situation and circumstances: mastering a skill well enough to solve a problem in one situation might not be enough in another situation. In a constantly changing society, the demands faced by an individual vary from one situation to another and from time to time. Therefore, in addition to possessing the specific basic skills for accomplishing a certain task, more flexible, generic and transferable competences

\(^4\) A specific working group has been established for language learning; for details, see http://www.europa.eu.int/comm/education/policies/2010/objectives_en.
are needed to provide the individual with a combination of skills, knowledge and attitudes that are appropriate to particular situations. For these reasons, many of the definitions describe rather the essential elements that comprise the competence and that are crucial as the competence develops from a basic level of mastery towards a more advanced mastery of the competence. The definitions thus leave room for judging the appropriate level of mastery of a competence with regard to the contextual factors involved.

Moreover, measurement of the mastery of most of these competences is so far limited. The existing measurement tools such as PISA and IALS give an indication of levels of mastery as regards literacy and numeracy. The Council of Europe’s Common European Framework of Reference for Languages (CEF) describes levels of mastery in foreign languages and research has been done to measure the ‘learning to learn’ competence. In addition, there are a number of national measurement tools for identifying the appropriate levels of mastery of basic skills in order to guide policymaking at various levels. While only some of the key competences are measurable, the framework helps to place these in the context of equally important generic and transversal competences that are more complicated to measure.

In accordance with the broader approach adopted by the working group on key competences, the overall definition of ‘key competence’ is as follows: Key competences represent a transferable, multifunctional package of knowledge, skills and attitudes that all individuals need for personal fulfillment and development, inclusion and employment. These should have been developed by the end of compulsory schooling or training, and should act as a foundation for further learning as part of lifelong learning.

The definition stresses that key competences should be transferable, and therefore applicable in many situations and contexts, and multifunctional, in that they can be used to achieve several objectives, to solve different kinds of problems and to accomplish different kinds of tasks. Key competences are a prerequisite for adequate personal performance in life, work and subsequent learning.

On the next pages the competencies for mathematical literacy have been described. These are copied from the European Commission Implementation of “Education and Training 2010” Work Programme (2004) (See also appendix 2)

5 A good example of this is the “digital literacy” competence. There are only few situations where basic skills in ICT are sufficient: in most cases effective use of ICT requires an appropriate level of critical thinking and a broader understanding of media.
### FRAMEWORK FOR KEY COMPETENCES IN A KNOWLEDGE-BASED SOCIETY

#### 3.1. Mathematical literacy

The competence consists of the following elements of knowledge, skills and attitudes as appropriate to the context:

<table>
<thead>
<tr>
<th>Definition of the competence</th>
<th>Knowledge</th>
<th>Skills</th>
<th>Attitudes</th>
</tr>
</thead>
</table>
| At the most basic level, mathematical literacy\(^6\) comprises the use of addition and subtraction, multiplication and division, percentages and ratios in mental and written computation for problem-solving purposes | Sound knowledge and understanding of numbers and measures and the ability to use them in a variety of everyday contexts is a foundation skill that comprises the basic computation methods and an understanding of elementary forms of mathematical presentation such as graphs, formulas and statistics. | Ability to apply the basic elements of mathematical literacy such as:  
- addition and subtraction;  
- multiplication and division;  
- percentages and ratios;  
- weights and measures to approach and solve problems in everyday life, e.g.:  
  - managing a household budget (equating income to expenditure, planning ahead, saving);  
  - shopping (comparing prices, understanding weights and measures, value for money);  
  - travel and leisure (relating distances to travel time; comparing currencies and prices) | - Readiness to overcome the ‘fear of numbers’.  
- Willingness to use numerical computation in order to solve problems in the course of day-to-day work and domestic life. |

---

\(^6\) Basic mathematical literacy ("numeracy") is a foundation skill for all subsequent learning in other domains of key competences.
FRAMEWORK FOR KEY COMPETENCES IN A KNOWLEDGE-BASED SOCIETY

3.1. Mathematical literacy (continued)

<table>
<thead>
<tr>
<th>Definition of the competence</th>
<th>Knowledge</th>
<th>Skills</th>
<th>Attitudes</th>
</tr>
</thead>
<tbody>
<tr>
<td>As mathematical competence develops further, it involves, as appropriate to the context, the ability and willingness to use mathematical modes of thought (logical and spatial thinking) and presentation (formulas, models, constructs, graphs/charts) which have universal application in explaining and describing reality</td>
<td>Sound knowledge of mathematical terms and concepts, including the most relevant theorems of geometry and algebra.</td>
<td>Ability to follow and assess chains of arguments, put forward by others, and to uncover the basic ideas in a given line of argument (especially a proof), etc.</td>
<td>Respect for truth as the basis of mathematical thinking.</td>
</tr>
<tr>
<td></td>
<td>Knowledge and understanding of the kinds of questions that mathematics may offer an answer to.</td>
<td>Being able to handle mathematical symbols and formulae, to decode and interpret mathematical language and to understand its relations to natural language. Ability to communicate in, with, and about mathematics.</td>
<td>Willingness to look for reasons to support one’s assertions.</td>
</tr>
<tr>
<td></td>
<td>Ability to think and reason mathematically (mastering mathematical modes of thought: abstracting and generalising where relevant to the question and modeling mathematically (i.e. analysing and building models) by using and applying existing models to questions posed.</td>
<td>Being able to understand and utilize (decode, interpret and distinguish between) different sorts of representations of mathematical objects, phenomena and situations, choosing and switching between representations as and when appropriate.</td>
<td>Willingness to accept or reject the opinions of others on the basis of valid (or invalid) reasons or proofs.</td>
</tr>
<tr>
<td></td>
<td>Ability to make use of aids and tools (including IT).</td>
<td>Disposition towards critical thinking; ability to distinguish between different kinds of mathematical statements (between e.g. an assertion and an assumption, etc.); understanding of mathematical proofs and the scope and limitations of a given concept.</td>
<td></td>
</tr>
</tbody>
</table>

7 Mathematics, although intrinsically linked to numeracy, is of higher complexity. “Mathematical behaviour” is about describing reality through constructs and processes which have universal application. It is best described as a combination of skills and attitudes. The definition emphasises the importance of “mathematical activity” and acknowledges the “links with reality” as a current emphasis in maths education.
What does this mean for the MiA project?

The aim of the MiA project is twofold. The first goal is to find out what competencies adults need to be able to manage mathematical situations in real life. For that the focus will be on the competencies as described for transferable and multifunctional knowledge and skills and for mathematical literacy, as described in the previous section. This will be done by studying relevant literature about numeracy and mathematical literacy in general, related to the MiA goals.

The second goal is to help teachers to find a way of teaching that supports their learners’ ways of learning the above competencies. This means that teachers reflect on their own ways of teaching and focus more on supporting and coaching of their adult learners. For this also some specific literature on this subject will be studied in the next chapter.

What does this mean for teachers?

To enable adults to develop the competencies they need in order to be able to manage mathematical real life situations, teachers will have to find a way of teaching and coaching that supports the learning of transferable competences, necessary to manage situations in which mathematics is embedded. Within this frame we will focus on the learning of functional mathematics, problem solving skills and learning skills in general.

The MiA project supports teachers in finding a way of teaching and coaching adult learners by creating learning situations in which adults can acquire the competencies they need for such new situations. These should be easy to apply in learning situations in school settings as well as in out-of-school situations. Since learning in practice almost always happens in the course of life, it is predictable that learning in practice often takes place through learning by doing. Adults come in situations they have to manage in their own ways and often without any help of others, certainly without the help of teachers. Therefore we may wonder whether teachers should teach or that they should try to create learning situations in which they can facilitate and support learning in order to help learners to find their best ways of learning and problem solving.

References

European Commission.
3. Theory in MiA

Mieke van Groenestijn

Introduction

Mathematical situations are almost always embedded in more complex real life situations that may require more than only mathematical skills. They often also concern language skills and problem solving skills. In addition adults need to have insight in their own best ways of learning to be able to process new knowledge and skills in the course of life, necessary to keep up with new technological developments. Therefore learning mathematics in real life situations is a complex matter and quite different from learning mathematics in school. Teachers in adult education need to realize that learning mathematics in a school setting includes more than only learning math. It also includes analyzing and managing situations in which math is embedded and that needs some “action”. Adults often have to decide what kind of “action” they have to undertake to respond to or to solve embedded “mathematical problems”.

The focus in this chapter is on how adults do and learn mathematics in real life situations. For that a few relevant theories have been discussed. This may give food for thought for teachers and will be the basis for the field experiments. The central issue is what it means to be numerate and how people become numerate in the course of life. This leads to a few research questions for teachers to reflect on. The following topics have been discussed:

- What does it mean to be numerate?
- Learning in practice
- Learning and teaching in adult education
- Transfer of information
- Acquiring and processing new information in real life situations
- Learning mathematics in real life situations and in adult education (six steps)

3.1 What does it mean to be numerate?

Over the past decades many discussions about numeracy, mathematical literacy and functional mathematics or mathematics for life took place. In fact there is little or no difference between these concepts, there is only a difference in “feeling” or in “application” when to use which definition.

The concept of numeracy was originally used in the broad meaning of feeling familiar with numbers in all kinds of everyday life situations (Cockcroft, 1982). This includes not only the four basic operations but also using numbers, fractions and decimals for measurement, ratio and proportions, percentages, dimension and geometry and even using basic concepts of statistics. A distinction appeared between mathematics and numeracy as a difference between school mathematics and functional mathematics for everyday life.
Mathematics was learned in school. Numeracy became the issue for adult education to make a distinction with school mathematics and to give mathematics a new meaning for adults. Numeracy includes the math that people need in their individual lives in real life situations. In this way adults who dropped out of school or developed math anxiety receive a second chance to acquire the mathematical skills they really needed and to overcome the problems they got in school. Numeracy is synonymous to functional mathematics. It has its source and focus in real life situations. Numeracy or functional mathematics can be based on informal and formal procedures.

Mathematical literacy, as is measured in the PISA study (OECD, 2006), is the result of compulsory and formal mathematics education. Such may include functional mathematics, of course, but in general it is based on school math and less or even not on actual real life mathematics. School mathematics is supposed to be applicable in real life situations, but that depends on what every individual person actually needs in his or her lived-in situation. It also depends on the capacity of the individual person to transfer the mathematical knowledge and skills he learned in school to actual real life situations.

The starting point in the MiA project is that adults experience mathematics in their individual real life situations and have to deal with that. Therefore a definition on numeracy should start from real life situations and should describe what it means to be numerate, referring to the competences adults need to be numerate. In some recent studies for adult education a few usable definitions have been described that are according to the notions of mathematical competences as described in the DeSeCo program.

In the international ALL study numeracy has been defined as “The knowledge and skills required to effectively manage and respond to the mathematical demands of diverse situations.” (Gal, van Groenestijn, Manly, Schmitt and Tout, 2002)

It is the opinion of the ALL numeracy team that “numeracy” itself cannot be tested, only “numerate behavior” can be observed. With this purpose in mind and to be able to create items for the numeracy assessment for the ALL survey, the team operationalized the definition as:

“Numbertate behavior involves managing a situation or solving a problem in a real context by responding to mathematical information that is represented in a range of ways and requires the activation of a range of enabling processes and behaviors.” (Gal et al, 2002)

Or more explicitly:

Numbertate behavior involves managing a situation or solving a problem in a real context (everyday life, work, societal, further learning) by responding (identifying, interpreting, acting upon, communicating about) to mathematical information (quantity & number, dimension & shape, pattern & relationships, data & chance, change) that is represented in a range of ways (objects & pictures, numbers & symbols, diagrams & maps, graphs, tables, texts, formulae) and requires the activation of a range of enabling processes and behaviors (mathematical knowledge and understanding, mathematical problem solving skills, literacy skills, beliefs and attitudes). (Gal et al, 1999; table 1).

By choosing one element from each of these five subcategories, one can compose a way of numerate behavior for each particular situation, for example: Numerate behavior involves managing a situation or solving a problem in everyday life by acting upon (estimation with money) to information concerning quantity and number that is represented by pictures (in advertisements in leaflets) and requires the activation of computational and estimation skills.
Through this way of describing numeracy the ALL definition comes very close to the mathematical competences of DeSeCo because it is applicable for almost every individual person in his or her individual situations.

A second definition about numeracy found its origin in adult vocational education, basic adult education and education at the workplace. Tine Wedege and Lena Lindenskov defined numeracy in the following way.

“Numeracy is a math-containing everyday competence that everyone, in principle, needs in any given society at any given time.” (Lindenskov & Wedege, 2001)

This definition is two-pronged:
- Numeracy consists of functional mathematical skills and understanding that in principle all people need to have.
- Numeracy changes in time and space along with social change and technological development.

A third definition that comes close to the MiA goal is from Diana Coben

“To be numerate means to be competent and comfortable with one’s judgements on whether to use mathematics in a particular situation and if so, what mathematics to use, how to do it, what degree of accuracy is appropriate, and what the answer means in relation to the context.” (Coben, 2000 p.10, (35)

The definitions above have two essential points in common: managing real life mathematical situations and the changes over time due to technological developments.

For the MiA project the following definition has been used as a work definition, derived from Van Groenestijn’s ‘A Gateway to Numeracy’ (2002):
Numeracy is a dynamic concept that can be defined as the individual’s competence to manage mathematical situations in everyday life, at work and in societal life and to accommodate flexibly to new demands in a continuously changing society.

3.2 Learning in practice

Adults have multiple tasks to fulfill in everyday life. They have families, are parents, neighbors, citizens, customers, consumers, employers or employees, patients, members of sport clubs, volunteers in organizations, etc. In all those situations they have their own specific roles, tasks and responsibilities which require an aggregate of integrated social, literal and mathematical competencies that they learn in the course of life. Knowledge and skills acquired in former school years are the necessary basis for this, but in general this is only a small part of what is actually needed to manage everyday life situations. Adults who have acquired only very basic skills in their former school years, and even adults with higher educational background but not in the areas that require a lot of mathematics, might experience difficulties to further develop the necessary mathematical skills they need to be able to organize and manage their daily life situations. When adults come back to school they often only want to learn what they need to learn and what is straight applicable in their daily

---

8 Parts of the text in this section are derived from “A Gateway to Numeracy”, chapters 5 and 7 (van Groenestijn, 2002).
lives. This requires a way of adaptive teaching that fits the needs and ways of learning of adult learners.

There is no specific theory about “learning in practice”, but several studies are done and have still been undertaken about the learning of adults in real life situations since the sixties in order to examine how adults develop knowledge and skills that are needed to function optimally in everyday life or to effectively manage all kinds of situations, in particular in work situations (among others: Lave, Murtaugh and De la Roche, 1984, Resnick 1987, Carraher, Carraher and Schliemann 1988, Lave 1988, Lave and Wenger 1991, Saxe 1991, Van der Kamp and Scheeren 1996, Noss and Hoyles 1996, Tuijnman, Kirsch and Wagner, 1997, Greeno, 1999). Several conclusions emerge from these studies. Some relevant main points for this study are discussed below.

1) Adults are free to learn. (Rogers, 1969). There is no compulsory education for adults. They learn because they need or want to be better informed, want to improve specific skills or to acquire more specific knowledge. They are not obliged to learn as they were in school. However, these days the need for lifelong learning becomes more evident due to developments in technology and continuous changes in society. In that frame adults are encouraged to learn in order to be able to keep up with these developments.

2) Learning happens in a functional situation. (Resnick, 1987) There is a need for learning. All that is done in real life is embedded in specific situations and every situation requires specific actions. Adults are continuously managing and solving problems and making decisions. Every situation is a source for learning but is also the context in which prior acquired knowledge and skills are applied. The advantage of learning in real life situations is that all new knowledge and skills are functional tools for managing and solving real life problems. One disadvantage may be that learning depends on the given situation. If there is no problem, then there is no need to learn. If there is a problem people have to solve that specific problem and this may be a source for learning, but the quality of learning and the result depend on the person in that particular situation. A second disadvantage may be that, because of the development of knowledge and skills in very specific situations, people might not be able to see links with other situations or to transfer this new knowledge and these new skills to similar, but perhaps a bit different, new situations. In this way situation-bound knowledge and skills are developed.

3) Learning in practice is characterized by learning through authentic materials. Whereas in school situations learning often takes place through text books, photos, schemes and with the help of artificial hands-on materials, in practice this can be done in the actual situation with authentic materials. For example: computing the area of a floor and determining how many tiles are necessary to cover the floor can be done with the dimensions of the actual floor and tiles and by using professional tools like a measurement tape. Such materials may make it easier for adults to understand the mathematical situation and to analyze and solve the problem. Real materials also often offer possibilities to solve problems in different ways. A real tile, for example, can be used to determine how many tiles fit in the length and the width of the floor, even without computing the actual area. Computations can often be done in a creative, informal way. Such cannot be done to the same extent with a photo of the tile and a map of the floor in a math book. Tasks in a text book often require application of formal computation procedures, like measuring the dimensions of both the tile in the photo and the map, computing the area of both and then dividing one by the other. Also, learning in a work situation is often related to learning how to work with specific tools,
e.g. operating specific machines like a paint mixer, a sawing-machine, computer directed machines. This “learning-by-doing” leads to “knowing-for-doing” and is the basis for functional numeracy. Boekaerts and Simons (1993) distinguish in this regard “knowledge-as-a-tool” and “knowledge-for-knowledge”. Whereas in school situations students often learn “subjects” because they should know it, in practice subjects are learned because people need it or want to know it, to be able do their jobs or other things. Knowledge acquired in practice is almost always functional and applicable.

4) Every learning situation is a socio-cultural determined situation. Saxe (1991) describes, referring to Vygotsky (1978), that social interactions are redirected by social and historical influences. These affect natural processes in cognitive development. In essence, learning is an interactive and social act in which everybody takes part. Communication by talking about problems which need to be solved and in what way, is an essential part of the learning process and a starting point of developing reasoning skills and problem solving strategies and skills.

5) Learning in practice focuses on “shared cognition”, rather than on “individual cognition” (Resnick, 1987). Though of course there are also situations in which people function individually, in work settings employees may often be complementary to one another, like a chief and a secretary, a nurse and a doctor, a car salesman and a technician. In many work settings people only have very specific tasks without having insight in the full production process of the product to be made, e.g. in the automobile industry. There are only a few people who need to have the overall overview on the entire production process. In other situations, e.g. in a garage, employees can help each other with problems that cannot be solved individually. In such situations people learn to ask questions, to discuss the problems they meet, to look jointly for solutions and to work cooperatively. Though students in school settings in recent years are more often expected to work together on problem-based and joint tasks, they are still assessed on what they can do individually. A student mostly passes or fails a test independent of the performances of other students. With the current changes in the adult education system, it should also be possible that students who work on joint tasks are assessed on cooperative learning aspects and study skills in addition to their individual competencies and their proficiencies in individual tasks.

6) The way in which learning in practice takes place is often via showing - imitating - participating and applying. There is no need to create specific instructional settings. People spontaneously work cooperatively when the situation requires to do so, like in work and family settings. In school settings we have to create such “practical” learning situations, often based on artificial instructional constructs in order to learn to work cooperatively. (Resnick, 1987)

7) For learning in practice people construct or re-construct their own “rules-of-thumb” and informal “rules and laws” for managing actions, situations, materials and the environment in which they work. For example to make concrete a rule-of-thumb is to use cement, gravel and sand in the ratio 1:2:3, but in certain situations this ratio may be adjusted. More informal rules and laws may emerge in working situations, e.g. after having completed a job, tools should be cleaned and put away and have their own places in the toolbox in order to prevent loss and damage. Such rules may develop as “generally accepted” rules, and by that they become part of common, general knowledge, but still situation-based and situation-bound. Such “rules and laws” are often developed in work settings to keep the work situation under control. General rules and laws learned in school will often deviate from those in practice. (Resnick, 1987). Situation-based rules and laws may affect the learning of adults in school.
situations as in adult education or in vocational education. At the same time the development goes towards higher demands for written documentation. Therefore it is an issue for adult education to support adults in making explicit “rules-of-thumb” and “rules and laws”, where in earlier times it was sufficient to keep them in practical, informal and tacit forms.

These key issues offer important findings for learning in adult education. In the current developments in adult and vocational education new learning environments can be noticed, created based on experiences and information from studies about learning in practice, e.g. problem based learning and cooperative learning. This is a good development, but teachers must realize that learning in school settings will never be the same as learning in practice.

In his article “Learning in and for Participation in Work and Society”, Greeno et al (1999) challenges the education systems for adults. He argues that learning for participation in work and society can only take place in the workplace setting or in social communities in order to give meaning to the learning. Learning in informal ways in the course of activity in a meaningful setting has shown to be much more effective than learning in classroom settings. Since education in other settings than people’s own familiar workplaces also offers advantages, the need for a balanced system of learning in and outside school situations for adults might be clear.

Based on Greeno’s studies (Greeno et al, 1999) the following three statements were discussed in the MiA project:

**Statement 1:**
Learning is fundamental to and a natural part of human activity.
Learning in a classroom setting is a not-natural situation.

Explanation: adults are used to learning in the course of life in informal and non-formal ways or in work situations. Learning in classroom settings is almost always conducted by a teacher and creates a formal teaching-learning situation or ‘academic’ way of learning in which adult learners may feel uncomfortable, for example because of negative school experiences in the past. In real-life situations adults can be teachers and learners in group situations, even at the same time when adults conduct their own learning processes.

**Statement 2:**
Activity, motivation and learning are not separable.

Explanation: when there is a need or a wish for learning then adults want to learn. If adults don’t see a need for learning or don’t see the need for learning a particular subject they have to learn, then they may not be motivated to learn.

**Statement 3:**
Adults don’t learn just in order to do but in order to become.

Explanation: adult learning is not just learning. It is more part of growing. Greeno et al discuss this topic related to “becoming” a (better) member of a particular community. There may be a distinction between learning for knowing and learning for doing. People want to learn things they can use “tomorrow” in their lived-in situations. By doing mathematics better in actual real life situations, people become better members of their communities.
Greeno et al define a community of practice as an aggregation of people who, through joint engagement in some enterprise, come to develop and share ways of doing things, ways of talking, beliefs, values – in short practices. When people see themselves accepted in a community as they are growing, they are challenged to learn more and to cooperate better in society. If they don’t feel accepted, they may feel discriminated. This stops the learning process and cooperative behavior. When people participate in a community then individual knowledge and expertise become shared knowledge and expertise.

These statements provide food for thought and may challenge teachers in adult education to reflect on why they teach, what they teach and how they teach in adult education. Learning in practice often means joint learning by showing and doing based on shared responsibilities, whereas learning in a school setting often means individual learning with the help of books, testing and doing individual exams. There are often only individual goals, no shared goals. Learning for adults must be meaningful, must make sense. Adults must know that what they learn in school they can use it tomorrow in their personal, societal and work situation.

3.3 Learning and Teaching in Adult Education

There is a gradual move in adult education from a pedagogical “teaching adults” into a more andragogical “helping adults learn” (Knowles, 1990). Dewey (1916) and Bruner (1968, 1996) already recognized that the human being is in principle a natural "learner". For Dewey, the human being is born with unlimited potential for growth and development, and education is one of the agents that facilitate growth (Jarvis, 1998, p.148). Bruner emphasizes that any didactical process, such as formalized instruction, is actually helping to create a sense of dependency in the learner rather than independency. (Jarvis, 1998, p. 146). Knowles (1990) in particular stresses people's own perspectives and self-responsibility as determinants for the learning process. In adult education we should prevent any dependency of adults on teachers and should emphasize the adults’ own competencies and potential for growth and development. Teachers in adult education are only “facilitators” of learning and should help adults learn to teach themselves. (Jarvis, 1998, Knowles, 1990, Brookfield, 1986, Goffree and Stroomberg, 1989). Adults should take responsibility for their own learning in school as they do so in their everyday life situations. These thoughts lay the foundation for adult learning in adult education and open the way for lifelong learning.

Paulo Freire

Freire’s theory became known as “Learning from Experiences” (Freire, 1970) and found its way into many countries all over the world. His keyword for literacy and adult learning was “dialogue”. When people come into dialogue they become aware of their own situation and that is the basis for improving their own situation. The only ones who can help to improve the situation they are living in are the people (whether individuals or peoples) themselves. Freire’s liberation education consists of acts of cognition, not of transfer of information. It is a learning situation in which the teacher becomes a learner and the learner is his own teacher.

“The teacher is no longer merely the-one-who-teaches, but the one who is himself taught in dialogue with the students, who in turn while being taught also teach” (Freire, 1970, pg.67).
Though Freire’s theory was developed in particular for people in oppressing countries, it has always been very popular in adult education in several countries, among other in The Netherlands. However, the way in which his ideas were elaborated and applied differs from the original way. It can even be discussed if his pedagogy is applicable in every culture. (see also Coben, O’Donoghue and Fitzsimons, 2000). During the seventies and eighties “learning from experiences” was applied as a way of instruction for adults, especially in adult basic education and in Open School learning centers, but it never resulted in a real system of learning and teaching in the Netherlands. Despite the poor implementation of Freire’s pedagogy in the Netherlands, his theory has contributed a lot to the development of the early literacy programs between 1970 and 1980. Teachers became aware of their own authority as a teacher and of their ways of teaching. This awareness was a start for teachers and learners in adult education onto mutual equality, respect, acceptance and critical citizenship as co-learners. His theory resulted in four starting points for the Dutch Adult Basic Education that started in 1987: (Ministerie van O en W, 1986, Act on Adult Basic Education)

- learners’ own decisions and self-management determining the actual program
- education and schooling in mutual relation
- learning from experiences
- mutual learning and teaching.

The gain of that time is that teachers and others in society started to look at their learners through different eyes. Illiteracy was something that could not exist in the Netherlands because everybody has had a chance to go to school. It was, and still is, hard to acknowledge that there were, and still are, nearly illiterate or semi-literate people in the Netherlands. In principle everybody has the right to learn and is supposed to have had ten years of compulsory education. Reading, writing and arithmetic difficulties are often ascribed to the person’s incapability or unwillingness to learn in former school years. The question whether the educational system or non-capable teachers might have caused learning problems in school is less emphasized, but perhaps not less true. In principle people are free and they can often manage their own lives in certain ways. They can come to classes for refresher courses, further development or for learning the Dutch language, whenever they want. This educational situation is not comparable with the oppressing situations Freire meant. However, the underlying thought of Freire’s theory could be a good basis for the development of an instruction model that would empower native and non-native illiterate and semi-literate citizens. Illiteracy and innumeracy still appear a hidden problem in western societies and should have much more attention and care. Freire’s ideas could also be a good way of education for specific minority groups within western cultures, like women who still live in masculine subcultures. His starting points are still valid and directive in the new system for adult education in the Netherlands.

In Spain Freire’s theory developed in a different way. At the AGORA Institute for Adult Education in Barcelona, for example, the starting points of Paulo Freire have been implemented in the learning situations. It is called “Dialogic Learning”. Dialogic learning is the methodology that is used in all workshops and classes. It consists of an egalitarian dialogue based on the argumentation and the cultural intelligence which is assumed that all people have developed in the course of life. Dialogic learning is based on solidarity, the equality of differences.

The interactive groups are a methodological innovation that has been taken upon maturity in the school. It includes that the participants can be learners and teachers in the same situation and at the same time. By this mutual learning and teaching everybody is involved as a teacher as well as a learner in the same learning/teaching process.
The seven principles of dialogic learning are derived from Flecha (2000):

1. Egalitarian dialogue
A dialogue is egalitarian when it takes different contributions into consideration based on the validity of their reasoning, instead of valuing them for the positions of power held by those who make them.
In egalitarian dialogue both students and teachers learn, since they all construct interpretations based on the contributions made. Nothing can be taken as definitively concluded, as assertions will always be subject to future analysis.

2. Cultural Intelligence
Everyone is capable of participating in egalitarian dialogue, although each person may demonstrate his or her ability in different environments. Those who perform better in the market or the factory may feel completely inhibited in the classroom; those who feel at ease in an academic milieu may be of no use at a neighborhood association meeting or in a discotheque.

3. Transformation
Dialogic learning transforms people's relationship to their environment. As Paulo Freire (1997/1995) says “as people we are not beings of adaptation but of transformation”.

4. Instrumental Dimension
Dialogic learning embraces every aspect of learning. It therefore deals with gaining all instrumental knowledge and skills considered necessary. Dialogic learning is not opposed to instrumental learning, but to the technocratic colonization of learning.

5. Meaning Creation
The energies and referents for that process are found in human beings themselves, in their relationships, in the dreams and feelings they constantly generate.
We can all dream and feel, give meaning to our lives. Each of our contributions are different and, therefore, irretrievable if not taken into account. Each excluded individual is an irreplaceable loss for the rest.

6. Solidarity
In order to foster solidarity you cannot hide behind eclecticism but must be willing to reject radically antisolidarist theories and practices. No one is neutral, particularly not those who claim to be. As Freire (1989) says: “it is not possible to be for someone without being against someone, who is against the one I am for”.

7. Equality of difference
Reforms in diversity have created educational inequalities. To overcome them, teaching needs to be reoriented in two ways: the aim of diversity should be changed to equality of differences, and the outdated conception of meaningful learning should be exchanged for that of dialogic learning.
3.4 Transfer of information

In general, transfer of knowledge and skills acquired in school to real life situations out of school always requires a mental action of the learner. School situations are never the same as real life situations. Learning in school often focuses on a formal way of individual learning by using books, quite often followed by doing tests, whilst learning in practice of happens through the process of showing – imitating – working together – doing it alone, all by yourself, without any test in the end. Cooperative learning has still been seen as something extra’s to also develop social skills, necessary to be able to work together in job situations, but still not as part of real learning. Learning in the actual situation is often the best way of meaningful learning, but such is not always possible.

To become aware of the kind of problems that may influence transfer of information and to find out what way of transfer would be best in adult education, the partners in the MiA project discussed some ways of transfer, posed by Paul Ernest (1998), at the Vilnius meeting in May 2005.

Paul Ernest describes four views on transfer of mathematical knowledge. (See appendix 3)

1. Applications Perspective:
   Transfer of learning is application: applying general knowledge in specific concrete situations through modeling
2. Cognitivist Perspective:
   Transfer of learning from one set or type of tasks to another – the transfer is disembedded knowledge
3. Problem Solving Perspective (constructivist):
   Transfer of learning from one situation to another through transport of personal transferable skills (with a person)
4. Situated Cognition Perspective (social theorists)
   Transfer of learning from one social context to another through the development of new capacities and facets of self.

The questions that were discussed by the MiA partners are:
What do we recognize of these descriptions in adult education?
Which of the four views comes close to your way of teaching?
What does it mean for learning and teaching in adult education?

During the discussion it appeared that the participating teachers found the problem solving perspective and the situated cognition perspective most relevant and recognizable for their own practices in adult education.

Most teachers agreed that for their learners’ learning in the actual lived-in situation would be best, but some learning centers only have the possibility to teach courses for certifications or diplomas. This means that there is no or little time for doing experiments on different ways of learning and teaching. Some teachers are involved in training on the job projects. They experience that their learners are motivated and know what they need and want to learn.

---

9 Issues for discussion during the second MiA meeting in Vilnius. Paper 2 (Mieke van Groenestijn, April-16-2005)
The four views on transfer of knowledge come from Paul Ernest: Mathematical Knowledge and Context. (1998)
Starting from that point, their learners may become interested in learning more about mathematics in general and for broader applications than what is only needed for their particular job. However, the problem with situated cognition is that it is often situated-bound. This may hamper transfer of knowledge and skills to other job situations. Adults can see this as only useful for a certain specific situation and less relevant for another situation.

From the problem solving perspective the list of key skills Ernest mentions, requires more than only learning mathematics. He mentions as personal transferable skills self-management, learning skills, communication skills, teamwork skills, problem solving and data-handling skills.

It is often assumed that adults develop these skills themselves in the course of life in all kinds of societal and job-related situations. In school people learn “mathematics” and teachers teach “mathematics”. However, when we think about mathematical competences, as in the way of the DeSeCo program, transferable skills are part of the key competencies and should be an essential part of learning in adult education. Teachers in the MiA project can see these transferable skills as integrated part of their programs, but in practice little attention is spent to that. It could be a challenge for the MiA project to find out when such personal transferable skills come up in learning situations and how to deal with that.

Based on the theoretical thoughts about learning in practice, the Greeno statements, Paulo Freire’s pedagogy and Ernest’s theory about the transfer of learning, the following research questions are posed for the MiA project for research in practice and for reflection.

Concerning learning:
1. Why do adults come back to school?
2. What do they want to learn?
3. How do they learn best?

Concerning teaching:
1. Why do we teach adults in adult learning centers?
2. What do we teach?
3. What can be the meaning of an adult learning center for learning in practice, in out-of-school situations?
4. How can we arrange a situation in which the adult learning center can be a center for transfer of learning in a school situation to learning in an out-of-school situation?

The main questions in this are:
1. How can we challenge adults to learn more about mathematics in out-of-school situations?
2. What role can an adult learning center play in supporting and coaching learning mathematics in out-of-school situations?

Many questions to be answered. It may be obvious that adults spend most of their time in real life situations and not in school settings. They have their responsibilities as adults and realize that they need to learn more for whatever reason, but most of the learning takes place in informal practical situations and not in school settings. When adults decide to go back to school they know that they have ‘things’ to learn that they cannot acquire without the help of others, but they will focus on knowledge and skills that they really need for their individual goals, e.g. to get a better position at work, in society or to be a better parent.
3.5 Acquiring and processing new information in real life situations.

Information in real life is often packed in sources like TV bulletins, newspapers, journals, texts, graphs, charts, tables, etc. Analyzing and understanding such information requires literacy and numeracy skills and notions of statistical concepts (Curry et al, 1996, Dossey, 1997, Gal, 1997, 2000). Adults are often supposed to acquire and process new information in their own informal ways, through “learning by doing” and through “learning by experiences”. However, when analyzing, in such processes the following details can be distinguished (Van Groenestijn, 2002):

1. read about, listen to or look at information
2. identify key points in the information
3. reflect on what is “new” to me?
4. communicate, discuss with others
5. reflect on possible implications for personal life. What does it mean to me?
6. reflect on possible implications for society or work.

If the new information concerns mathematical or statistical issues then the adult needs to have acquired some basic mathematical concepts on which they can build to be able to understand and process the new mathematical information.

In adult education adult learners can be prepared for acquiring new knowledge in real life situations by offering them opportunities to acquire the skills needed how to process such information. Discussion with other learners is an essential part of this learning process. Topics for discussion could be provided by the learners themselves but should also be incorporated in the program. Topics can be discussed in small subgroups, but to be able to do so effectively, guidelines are desirable. It cannot just be assumed that learners themselves can develop guidelines for how to work in such situations. Central to such discussions are mathematical and statistical reasoning that finds the logic behind the information and helps explain information to others. This means that learners must be open for discussion and critical comments. Essential in acquiring new information is that learners learn to ask questions to get things clear, or go find more information about a certain detail. For that they need to know how to access the internet, libraries, dictionaries and other resources. Learning centers can help to arrange access to such resources.

3.6 Learning mathematics in real life situations and in adult education.

Teachers in adult education are often confronted with adults who say that they don’t need mathematics in everyday life and by this they mean the mathematics they learned in school from textbooks. In practice, however, they do mathematics in all kinds of situations, most of the time in informal ways or in specific situations at work. They often even don’t realize that they do something with mathematics, they just do it, for example when they cook a meal for the family, drive a car, read a time schedule, paint a house, do some gardening, play football or tennis or go for shopping. In fact, they manage mathematical situations all day long.

For adult education it means that the actual real life situations are the source as well as the focus of learning mathematics. Learning starts in the actual lived-in situation of adults and aims to develop mathematical knowledge and skills that are usable and applicable in these situations. But at the same time it also aims to enable adults to broaden their perspectives by
which they develop competencies for better functioning in their actual lived-in situations and for further learning. Learning mathematics in adult education aims to lead to functional numeracy.

This starting point has consequences for the way in which learning mathematics in adult education is organized. In an ideal situation learning mathematics is organized in authentic situations, for example on the work floor. In most situations, however, it is organized in school settings. The challenging question for the MiA project is how to organize learning mathematics in adult education in such a way that the gap between school and real life will be bridged.

Learning mathematics in actual real life situations is often experienced as problem solving. When it comes to analysis of such situations we may distinguish the following details:

- There is a situation to manage or a problem to solve
- locate the situation or the problem as a mathematical situation,
  (it has something to do with numbers)
- identify the mathematics in it;
- analyze and structure the mathematical information in it
- interpret, give meaning to the mathematical information
- plan, discuss possible steps for solving the problem
- choose a solving procedure
- do computations, if needed, or act otherwise
- check the result
- apply contextual judgement, if necessary
- check possible consequences
- make decision
- reflect on the process.

To recognize the way adults learn in practical situations the steps for acquiring and processing new information must be combined with the steps for managing mathematical situations and solving real life mathematical problems. The above means that learning processes in adult education should be organized in the best possible ‘authentic’ situation, an actual learning situation. This can be done through the following steps:

1) Bring the learner in a potential mathematical situation
Such a situation could be ‘sales’, for example. The teacher knows that they may encounter a mathematical problem in the situation. The teacher organizes a ‘sales’ situation by, for example:
- bringing the learners in an actual authentic situation, e.g. in department store or street market
- asking them to tell a story about their experiences concerning sales
- showing something with a discount price (either the learners or the teachers)
  e.g. show a coat priced 150 euro with a label: 15% off

2) Identify problems in the situation
Focus or zoom in on mathematical problems, e.g. the learners says: “I don’t how to compute the new price. I just pay the amount at the cashier desk they ask me to pay”
3) Plan the problem solving procedure
The teacher challenges the learners to solve the problem: “How do you think you can solve the problem?”
Learners may find all kinds of informal and formal problem solving procedures.

The teacher’s task is to interact with the learners and try to discover what learners know and can do and what they don’t know or do wrong, e.g. the learner states that 10% reduction is always 10 euro off.

4) Do the problem solving
At this point the learning process can start, e.g. by discussions among learners (interaction)
Try to connect with the learners pre-knowledge and good conceptions.
e.g. the learner knows that 50% is half. How would you go on?

5) Check the result
Can the learners explain why their answers or solutions are correct or not?

6) Review the process. What did the learner learn?
The learners discuss what they learned. What is new to me? What does it mean for me in my personal life or for me in my work situation?

These six steps can be applied in every actual real life situation. They can help teachers to create an almost real-life situation in school settings. When teachers and learners are aware of these six steps then both, teachers and learners, are more and better actual involved in the learning process itself. It may help the learners to discover what they already know and what they really need and want to learn. It may also help the teachers to find out how they can support and coach their adult learners in such a way that adults feel independent and can organize their own learning processes. In this way Freire’s pedagogy and the general starting points about adult learning are recognized.

Along the lines mentioned above the teachers in the MiA project experimented in their own practices with the six steps. (see next chapter)
References


Curry, Donna, Mary Jane Schmitt and Sally Waldron (1996) *A Framework for Adult Numeracy Standards: The Mathematical Skills and Abilities Adults Need to be Equipped for the Future.* The Adult Numeracy Practitioners Network, funded by the National Institute for Literacy


ISBN 1-85853-083-0


Freire, Paulo (1970) *Pedagogy of the Oppressed* New York, Herder and Herder


Sponsored by the Organization for Economic Cooperation and Development and U.S. Department of Education
Groenestijn, Mieke van (2002). *A Gateway to Numeracy. A Study of Numeracy in Adult Basic Education*
CD β Press, Centrum voor Didactiek van Wiskunde, Universiteit Utrecht.
London, New York, Routledge
[Functional literacy- and numeracy skills of older adults in the Netherlands]
(Report of IALS results in the Netherlands in particular regarding older adults).
Amsterdam, Max Goote Kenniscentrum, University of Amsterdam
Knowles, Malcolm, 1990, *The Adult Learner, a neglected species*
 Houston, London, Paris, Zürich, Tokyo, Gulf Publishing, USA
 In: Rogoff, B. and Lave, J. (ed) *Everyday Cognition* (p. 67-95)
Cambridge, Harvard University Press, England
Lave, J (1988a). *Cognition in Practice*
Cambridge, Cambridge University Press
Cambridge, Cambridge University Press
Lindeskov, L. and T. Wedege (2001). *Numeracy as an Analytical Tool in Mathematics Education and Research.*
Centre for Research in Learning Mathematics, Roskilde University, IMFUFA. (publication nr. 31)
ISSN no. 1600-2472
Netherlands
Dordrecht, Kluwer Academic Publishers
 In: *Educational Researcher, 16,* 13-20
Westerville, Ohio: Merrill
Hilsdale, New Jersey, Lawrence Erlbaum Associates, Publishers
Cresskill, New Jersey, Hampton Press, Inc.
Cambridge, MA: Harvard University Press
4. Field Experiments

Introduction

During the project teachers in MiA shared their experiences with learning mathematics by adults in their own practices. In the first MiA meeting in Vilnius teachers from Denmark, Netherlands and Spain presented their ways of working with adults in different settings and based on different starting points. In these presentations some of the theories in chapter 3 have been processed. A summary of these presentations is given in section 4.1.

During the meeting in Barcelona, teachers shared their thoughts about “good practices” of learning and teaching in adult education. A summary of this is given in section 4.2.

Also during this meeting the “six steps”, as described in chapter 3, were introduced. After that meeting the teachers experimented in their own countries with this way of working. The results from Denmark, Spain, Hungary and the Netherlands are elaborated in section 4.3.

4.1 The first experiences

In Denmark a curriculum called the Preparatory Adult Education (PAE), starting 2000, is sometimes offered at the workplace. The curriculum involves activities of counting, measuring, locating, designing, playing, explaining (Bishop, 1991), involves authentic media and data and involves mathematical concepts and operations. Development of, so called, learners’ mathematical awareness is stressed.

The starting points are that
- Numeracy is relevant for participation in adult education and training, for keeping and improving job, for societal participation and for everyday life (see chapter 2). Adults with low skills hesitate to enrol in courses, but often adults are more skilful than they think themselves. Adult learning centres must help adults to recognise their skills.
- Adults learn better when it is meaningful for them to learn, in particular when activities and materials are authentic. (see chapter 3)
- Mathematics learning can cause difficulties, even for motivated adults.
- Adults prefer different ways of learning and only through dialogue the teachers learn more about their adult learners. But for all adults’ experiences from learning in practice outside school can support their learning in the PAE courses, and what they learn in the PAE courses can qualify the adults’ action in their everyday life.

At Adult Learning Centre Fyn – Glamsbjerg courses are planned in cooperation with the workplace and then the teachers get information about the everyday work at the workplace and can use practical mathematics examples from the workplace in the course. To ensure that courses at the workplace also serve broader demands than company demands, the teacher can mix the workplace examples with examples from personal life and citizen life. One advantage from facilitating numeracy learning at the workplace is that the adult learners are in an...
environment where they feel at home. This contributes to enhance a more balanced and more egalitarian relationship between teacher and learner compared to what it often is in a school setting. The Danish system is flexible. Whenever a group of learners is ready to start a course, in principle there are teachers ready to teach the course. In addition, exams are being arranged many times a year. The possibility to do the exam (something they don’t necessarily have to do) gives some of the learners a great deal of motivation.

According to the Dutch law, adult immigrants in The Netherlands in the age group 20 to 50 years, are obliged to do a special introduction course. It includes: learning Dutch and learning about Dutch cultural society in order to enhance the possibility of functioning in society through working and participating in vocational education or in different kinds of social activities. After about a year, the students reach language level 1 and they have started a portfolio with a personal action plan (PAP). At this stage, the students’ Dutch language skills are regarded as being sufficient to start the numeracy program. From that moment, all activities for each student within and outside school will depend on the kind of intentions the students have for the near future. For instance in a Hotel & Catering course (level 1) the learners spend two days a week on the work floor where they practice what they are being trained for. Sometimes, the teacher visits the work floor to learn about the real life situations where the students face numeracy events. As part of the weekly math-session (2.5 hours), the teacher in Utrecht (ROC-MN) does “Integrated Mathematical Action” (IMA) around an actual real-life situation with the students structured along ‘the six steps’ of problem solving (See chapter 3). The assumption is that, when embedded mathematics in actual real-life situations are explicated in mathematical contexts, the learners wishes to develop their numeracy skills will be encouraged.

In AGORA – Association of Participants in Barcelona mathematics is offered to adults aging from 18 to 82 years. Examples from real-life situations are the basis for numeracy activities. Within the group the learners can discuss the problem in order to give better opportunity to learn more about the problem. A very important side effect from using this method is the opportunity of giving someone within the group the chance to explain how to solve the problem. The teacher – learner relationship is based on dialogue. When dialogue between the learners and the teacher is equal, the teacher has the opportunity to learn about the learners’ solving strategies and to achieve a deeper understanding about the problems the learners have. This will help the teacher to guide the learners through the cognitive process of learning numeracy. All learning and teaching in AGORA is based on the seven starting points in “Dialogic learning” (see chapter 3). The adult learners can actively participate in all types of activities in AGORA, also in activities that only learners engage in other educational institutions. Materials used in AGORA for learning mathematics are developed in cooperation with adult learners. Also in every MiA seminar from AGORA both learners and teachers have participated.

All together the experiences from the Danish, Dutch and Spanish partners demonstrate that learning takes place in a social context. All three partners pay much attention to how learners’ personal experiences can be utilized in the teaching/learning process.
4.2 MiA teachers’ views on good practices

To MiA teachers good practice means:

The teachers facilitate and coach adult learners’ learning.
Teachers focus on how to best stimulate the learners, how to best ask questions and how to best summarise learners’ work.

Teachers are aware of and reflect upon how they best support that adult learners come up with own examples, tell stories and explain material.

Teachers are aware of and reflect upon how they best pose their questions to the adult learners.

Teachers are aware of and reflect upon how they best support adult learners to report on their findings. Good practice in MiA is that teachers are aware and reflect upon how they best react when adult learners working independently either individually or in groups reach a wrong solution.

The attitude of teachers has to be respectful towards the learners. They listen to the learners and give them the opportunity to experience that they can do more and know more than they often think they can.

Teachers can pose questions like ‘Where do you think you need math in your daily life?’ and ‘Where do you experience that some mathematics is a problem for you?’, ‘Where do you need mathematics to solve a problem?’, ‘Where is it not a problem for you?’ and ‘Where does mathematics help you?’.

Teachers can ask the learners about situations they have experienced, what they think they understand and do not understand, and what they would like to know and be able to do.

Teachers can ask learners to try to work on some of their own problems, let them themselves summarize what they have done, and let them see how differently they solved it. Teachers can ask the learners to try to do it in another way than they use to do, and then let them explain to each other how you did. Teachers can ask learners to slightly change the context or the figures, and let them solve it again and explain what they found out.

Teachers can support learners by having tell them to each other how they do and how to do computations in specific situations. That does not mean that teachers can never explain anything. Teachers can support learners to develop formal ways of doing computations or by showing formal ways to the learners and talk about how they relate to learners methods. Teachers should not say ‘you must do it this way!’

Teachers can ask learners to describe a situation and ask if they see any mathematics in the situation. When some learners know a lot, others know a little or nothing, then the learners can discuss among themselves how they will go on, they can choose one of the suggestions and try out as a start, and everybody has to come to a kind of understanding. Teachers can best support by e.g. suggesting how to go on, if the group is stuck, by saying some positive things as ‘please go on’, and if they know that they are not on the right track, they could...
suggest that they find the consequences. Then the learners maybe will go back and choose a different way, and if they don’t understand the teacher may give some small hints.

The teacher ensures that the learners reflect on what they have been doing. He can go through the process step by step in dialogue with the group. The teacher assures that everybody participates. The teacher support the learners to try out similar problems in a different context or with other figures and measures. Good practice means that the teacher invites the learners to participate in a try to generalise their findings.

Teachers are aware of and reflect upon how to best involve the learners in deciding and presenting a situation to work on. Learners may think that they are not allowed to come up with their own situations and problems. One idea is to ask the learners to tell about what they did the day before. The teacher could start telling about his own experiences the day before as inspiration. Another idea is to stimulate the learners to make an investigation about what can be mathematics and mathematical problems in situations in their life. And finally, let the learners tell stories and then the teacher suggests in what way mathematics come into play. Such examples can stimulate the learners to find examples themselves.

The teacher analyses situations together with the students to formulate what actually is the problem in the situation, which information do you need, what is relevant to know about, what is not relevant to know about, which things in the situation do you understand and not understand, and do you think it looks like something you know from other situations? From the situation analysis specific problems can be formulated. Teachers ask the learners to find different ways of solutions and assure that all learners can see possible ways of solutions so everyone can choose what they prefer. Then teachers or learners present a slightly different problem to be solved with the method the learners prefer. Teachers invite the learners to wonder about which consequences can be taken by persons or society according to the results of the problem solving.

It is not necessary that the teacher always knows the answers. He finds the topic interesting, can participate in discussions and is curious to learn.

Teachers realize that the same situations do not interest all people. When you win an amount of money, you are interested in how to use the money and how to save and invest. When in a shop you can calculate 50% and 25% discount, but need to calculate 20% without being able to do it, then you are interested to learn to calculate 20%. When you wish to make scarves as Christmas gifts and have some fabrics 1.2 meter times 6 meter, you are interested to know what kinds of scarves are possible to make.

To investigate good practices in depth and to describe and analyse examples from practice the MiA teachers offer field experiments. These experiments focus on real life situations and the use of the six steps to manage these situations. Afterwards learners and teachers can discuss their own way of doing mathematics in such situations along the format of six steps. (see next section).
Denmark
You and your body
Food-Energy

If you eat and drink exactly as much energy as your body needs, then you won’t become thinner or fatter. You use energy if you use your body. It’s easier for you to keep the ideal weight if you move a lot every day.

We normally measure food-energy in kilo-joule (kJ) or calorie. When you buy food or drinks, you can see on the container how much kJ it consumes per 100 grams.

A common man consumes about 11000 kJ in 24 hours. A common woman consumes about 9000 kJ in 24 hours.
Discuss:
- Did you know that there was so much energy in food?
- What surprised you most?
- What did satiated you most?
- What happens if you get sufficient energy?

Exercise:
- How much energy is in food and drinks that you use?
- Look at four food-containers at home!
- How many kJ are in the food per 100 gram?
- Weigh some food – how many kJ are in it?

<table>
<thead>
<tr>
<th>Product</th>
<th>kJ per 100 gram</th>
<th>kJ per gram</th>
<th>Weight of the product</th>
<th>kJ in the product</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oil of olives</td>
<td>3.700</td>
<td>3.700/100=37</td>
<td>500 gram</td>
<td>37x500=18.500</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

How long does it take to burn energy?

Discuss:
- What shows the table below?
- Why doesn’t it take time for the body to burn water?
- Why takes it longer time to burn an apple when you watch television compared to when you walk?
- Which kind of food takes longest time to burn?
- What surprises you most?

<table>
<thead>
<tr>
<th></th>
<th>Rest</th>
<th>Easy activity</th>
<th>Normal activity</th>
<th>Hard activity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kJ</td>
<td>Relaxing, look TV</td>
<td>Work at PC, writing</td>
<td>Cycling, strong, running</td>
</tr>
<tr>
<td>Water</td>
<td>0</td>
<td>0 min</td>
<td>0 min</td>
<td>0 min</td>
</tr>
<tr>
<td>Gulerod</td>
<td>120</td>
<td>25 min</td>
<td>16 min</td>
<td>6 min</td>
</tr>
<tr>
<td>Apple</td>
<td>245</td>
<td>50 min</td>
<td>33 min</td>
<td>12 min</td>
</tr>
<tr>
<td>½ lt. Soft drinks</td>
<td>850</td>
<td>2 h 54 min</td>
<td>1 h 16 min</td>
<td>43 min</td>
</tr>
<tr>
<td>Snickers bar</td>
<td>1250</td>
<td>4 h 15 min</td>
<td>2 h 50 min</td>
<td>1 h 3 min</td>
</tr>
</tbody>
</table>
Body Mass Index (BMI).

BMI is a measure to indicate your size. You can compute BMI in the following way:

\[
\text{BMI} = \frac{\text{weight in kg}}{\text{height in meters} \times \text{height in meters}}
\]

Example: A person’s weight is 80 kg and his height is 1 meter and 75 cm. What is the person’s BMI’?

\[
\text{BMI} = \frac{80}{1,75 \times 1,75} = 26,1
\]

Instead of writing \(1,75 \times 1,75\) we can write \(1,75^2\)

Another person’s weight is 55 kg and her height is 1,62 m. The person’s BMI is

\[
\text{BMI} = \frac{55}{1,62^2} = 21,0
\]

The first person has a little over-weight, but the second person has a good BMI. A perfect BMI is between 20 and 25.

Exercise:

Find the BMI for

a) A person measuring 2,00 m and weighing 100 kg.
b) A person measuring 1,85 m and weighing 75 kg.
c) Your own BMI

Steptest.

Now it is time to be active
Stop doing math.

On the website www.steptest.dk you can do your fitness test.
You and your body

Participants: 12 students, FVU, AVU and HF, age 20-60, 9 women and 3 men.
Teacher: JKT
Place: VUC FYN Glamsbjerg as a part of an introduction course.

Step 1:
Context: Bring the learners in a potential mathematical situation
These years there a lot of focus on food, motion and how to eat what you want – and still have a normal weight.
We have a talk about food – energy in food etc. – and the students bring different kind of food-package to the next lesson.

Step 2:
Identify problems in the situation and focus/zoom in our mathematical problems
- What is the difference between grams and kilo grams?
- What is the difference between J and kJ?
- What is Joule?
- Formula to calculate the body’s structure?

Step 3-5:
Planning for solving the problems
Problem solving procedure
Check the results
See the materials below

Step 6:
Review the process
Some of the students learned more about grams and kilograms.
The students learned a lot about J, kJ and energy in food.
Some of the students learned to use a formula
Favourite dish

Participants: 14 students, FVU, age 18 – 58, 10 women and 4 men, 13 Danes and 1 woman from the Philippines.
Teacher: Ninna Jepsen
Place: VUC Fyn Glamsbjerg as a part of a FVU-course

**Step 1:**
*Context: Bring the learner in a potential mathematical situation*
The students very often find it difficult to decide what they are going to make for dinner. I asked them why they didn’t ask some of the others. What did they have for dinner yesterday evening?
Then you can decide what dish you want to try.
We decided to bring our favourite recipe for the next lessons.

**Step 2:**
*Identify problems in the situation and focus/zoom in on mathematical problems*
- what is the difference between grams and kilograms?
- what is the difference between decilitres and litres?
- what do I do when the recipe is made for 6 persons and we only are with 2 persons?

**Step 3:**
*Planning for solving the problems*
Exercises changing grams into kilogram and the opposite.
Exercises changing decilitres into litres and the opposite.
Exercises changing recipes from 2 persons to 4 persons, 3 persons to 6 persons, 2 persons to 5 persons, 4 persons to 2 persons, 6 persons to 3 persons, 5 persons to 2 persons.
In general change the recipe from X persons to Y persons

**Step 4:**
*Problem solving procedure*
Choose some of the recipes and change them for your family

**Step 5:**
*Check the results*
The students told the others what they did in order to remember how to turn grams into kilograms, decilitres into litres and how to change a recipe for a certain number of persons.

**Step 6:**
*Review the process*
Some of the students learned how to change grams into kilograms.
Some of the students learned how to change decilitres into litres.
All students learned to adjust a recipe to their own family.
Hungary
Report of the MiA learning experiment in Hungary

Background to all three fieldwork tasks

Teacher: Zsuzsanna Selymes, Balassagyarmat

The MIA fieldwork in Hungary took place with inmates in a prison in Balassagyarmat between 01 December 2005 and 31 March 2006. The inmates are attending a course to finish their education on the 10th year level, i.e. secondary school 2nd year (normally done at age 16). However, it is a heterogeneous group, their level of mathematical knowledge is lower than in normal schools. Depending on their learning programme, they have 2-3 hours of maths at one session, however, it can happen that they have only one session to study maths in a month. The participants of the course were 11 men aged between 22-45, 8 persons of Roma ethnic origin.

<table>
<thead>
<tr>
<th>Name</th>
<th>Age</th>
<th>Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Róbert T.</td>
<td>38</td>
<td>poor</td>
</tr>
<tr>
<td>Róbert R.</td>
<td>31</td>
<td>good</td>
</tr>
<tr>
<td>Róbert S.</td>
<td>37</td>
<td>medium</td>
</tr>
<tr>
<td>István S.</td>
<td>35</td>
<td>poor</td>
</tr>
<tr>
<td>Géza Cs.</td>
<td>45</td>
<td>good</td>
</tr>
<tr>
<td>János K.</td>
<td>27</td>
<td>good</td>
</tr>
<tr>
<td>Endre Sz.</td>
<td>27</td>
<td>good</td>
</tr>
<tr>
<td>Olivér R.</td>
<td>26</td>
<td>good</td>
</tr>
<tr>
<td>László O.</td>
<td>25</td>
<td>poor</td>
</tr>
<tr>
<td>Sándor G.</td>
<td>22</td>
<td>good</td>
</tr>
<tr>
<td>István Sz.</td>
<td>24</td>
<td>medium</td>
</tr>
</tbody>
</table>

Some of the participants have already heard about the MiA project because they participated in trying out one of the good practice examples from the project. It was the learners’ request to use EUROs in the mathematical tasks because they found it interesting that they are participating in a European project.

Because the course is taking place in prison, there are some limitations to bringing learners into a concrete mathematical situation. Through discussions with the participants, the teacher found out what could be the mathematical problems that are of interest to them. All the men are in contact with their families and know about the money problems their families have. After the discussions the teacher decided to choose 3 tasks: family budget, sales and health, all related to money issues.

Three groups were created and each group got one of the tasks at one occasion. So the three groups were working on three different tasks at the same time. On a rotating system, on other two occasions the groups did all the three tasks. There were three sessions altogether 1 teaching hour each.
The groups received the tasks printed on a sheet of paper. Working in the group involved reading the task aloud, reading again, understanding together, listening to each other, making suggestions to find the solution, making arguments and conclusions, drawing and calculating. When all the tasks have been done, the solutions were discussed in the whole group. With the teacher’s guidance, drawings were made on the blackboard and the groups shared their suggestions for the solution. Finally the correct solution was agreed on.

**Task 1 Family Budget**

<table>
<thead>
<tr>
<th>1) mathematical situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>The income of your family is 180 euros a month. 40% is spent on bills, 1/3 is spent on food, 10% is spent on clothes and 20 euros is spent on cleaning materials (soap, washing powder etc). How much money is left to spend on other things?</td>
</tr>
</tbody>
</table>

(Comment: 180 euro is the equivalent of the minimal wage in Hungary.)

<table>
<thead>
<tr>
<th>2) identifying problems in the situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>The task consisted of several parts, and the mathematical problem was understood after reading three times. They soon realized that 40% and 10% comes to half of the sum. Then they started to calculate 1/3 of the remainder. They only realized that they had misunderstood the text when they checked their calculations by drawing. They had a lot of discussion about the reality of the figures they received, they thought that 20 Euro were too much for cleaning materials and what was left to spend on other things was too little. They agreed about the reality of the sums they had to spend on bills and clothes, they knew how expensive they were.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3) planning the problem solving procedure:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculating half of the family income caused no problem, they did not bother to calculate 40% and 10% separately. It was a problem to calculate 1/3 because it was not clear to them what is left or 1/3 of the total. Some of them wanted to divide by 3 what is left, others wanted divide the total. The teacher asked them to read the text again and make a drawing of the problem.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4) Problem solving procedure:</th>
</tr>
</thead>
<tbody>
<tr>
<td>The learners had to calculate several sums to get the correct answer/solution. They are afraid of tasks like this because they don’t like calculating with percentages. However, they could avoid it here by applying half instead. Comparing fractions (1/2 and 1/3) was too difficult for them and they could not do it. It was also difficult to define how much is 1/3. Making a drawing helped them. It was unusual for them to work with a family budget, because they found it strange that one can calculate in advance what will be left. In their opinion “You can never know, how much is left.” Drawing conclusions proved to be difficult too. They needed some guidance after they calculated the different sums to add up the results and subtract it from the total to get how much is left to spend on other things. Several learners suggested that the sums should be</td>
</tr>
</tbody>
</table>
subtracted one by one so that they should know if they still have money. The procedures of the three groups differed in the sequence of making the calculations and in their drawings. The calculation of percentages was done by drawing conclusions by two groups and by reading the result from their drawing by one group (Group I). This group made drawings of money cards. The number line was not really a proper one but it functioned well. Dividing the pie chart into 1/3rds was not successful.

Comparison of the work of the 3 groups

<table>
<thead>
<tr>
<th>Aspects</th>
<th>Group I</th>
<th>Group II</th>
<th>Group III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Illustration</td>
<td>money cards</td>
<td>number line</td>
<td>pie chart</td>
</tr>
<tr>
<td>Procedure</td>
<td>40%+10%=half 1/3&lt;sup&gt;rd&lt;/sup&gt; of the remainder is calculated first, correction is made after guidance</td>
<td>Drawing conclusions: 1 % is calculated first, then the value of 10% and 40% respectively</td>
<td>50 %=half, but the calculation and the drawing are incorrect. Then 1% is calculated and the different parts separately. 1/3&lt;sup&gt;rd&lt;/sup&gt; of the remainder is calculated initially.</td>
</tr>
<tr>
<td>Checking the results</td>
<td>Making a drawing, dividing the money cards after teachers explanation</td>
<td>Making a drawing, which is clumsy, but correct</td>
<td>Is done only orally, using addition is considered to be enough.</td>
</tr>
<tr>
<td>Math level of group members</td>
<td>Good-medium</td>
<td>Medium-poor</td>
<td>Poor</td>
</tr>
</tbody>
</table>
1. Feladat:
családi költségvetés

Egy család bevétele 180 Euró havonta.
Ennek 40 %-át rezsiire, 1/3 részét élelmiszerre, 10%-át a gyerekek ruházatára és
20 Eurót tiszttősözvekrek költenek. Mennyi pénz költethető egyébekre?

180:400 = 18
18:40
72
72:40
72%
180:1/3 =
52
52

10|10|10|10|10|10|10|10|10|10|10|10|10|10|10|10

10 Euró költethető mértéken

20 (40%+10% = 50%)

Group 1
Egy család bevétele 180 Euró havonta.
Ennek 40 %-át rezsire, 1/3 részét élelmiszere, 10%-át a gyerekek ruházatára és 20 Eurót tisztítószerekre költenek. Mennyi pénz költhető egyebekre?

\[
\begin{align*}
100\% & \rightarrow 180 \\
10\% & \rightarrow 18 \\
10\% & \rightarrow 18 \cdot 0.10 = 1.8 \\
180 \cdot \frac{1}{2} - 30 & = 60 \\
100\% & \rightarrow 180 \\
1\% & \rightarrow 1.8 \\
10\% & \rightarrow 18 \cdot 1.8 = 18 \\
18 \cdot 10 & = 180 \\
20 & = 20 \\
10 \text{ Euro nemadó}.
\end{align*}
\]

Group 2
1. Feladat:
 családi költségvetés

Egy család bevétele 180 Euró havonta.
Ennek 40%-át rezsire, 1/3 részét élelmiszerre, 10%-át a gyerekek ruházatára és
20 Eurót tiszttőszerekre költene. Mennyi pénz költhető egyebekre?

Bevétel: 180 euró havonta

10% - a 18 euró

40% = mert 20 euró - 10 euró mand él, 70 euró

Egyebek

Családi próbállomás

45

40%

100% → 120 euró

1% → 180 : 100 = 1,8

40% → 40 euró

Group 3
Task 2 Sales

1) mathematical situation

Sales
A pair of boots costs 60 euro. In the winter sales there is a 30% discount on this pair of boots. How much will you have to pay for it? How much will you save?

2) identifying problems in the situation

The learners identified the following problems:
What could be the real price of the boots?
Why are they cheaper?
Will I not be cheated?
Should I buy them, winter is soon over?
If the discount is 30%, why do I not pay this sum?

3) planning the problem solving procedure

At the beginning one learner tries to go around the problem and says that 30% is 1/3 and it is not important to calculate correctly. He tries to convince the others. Somebody else suggests calculating 1% first, somebody else says that it is easier to calculate 10% and multiply that by 3.

4) Problem solving procedure:

Group I: 1% is calculated first and then 30%. When they get this result, it is mistaken as the new price, they like the low price. They have to be warned that this is not the final result and work on how much is to be paid.
Group II: They know that what is left after subtracting the discount should be the answer and they use the decimal fraction of 70% in their calculation from the beginning.
Group III: They use 1% to calculate 30%, this is drawn on the pie chart and correct answer is given.
Comparison of the work of the 3 groups

<table>
<thead>
<tr>
<th>Aspects</th>
<th>Group I</th>
<th>Group II</th>
<th>Group III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drawing</td>
<td>-</td>
<td>-</td>
<td>Pie chart</td>
</tr>
<tr>
<td>Procedure</td>
<td>1% is calculated first, then 30 %, then subtraction is done</td>
<td>70 % is calculated using the decimal fraction of 70 %</td>
<td>Drawing is made, then corrections are made and the connection between the fraction and percentage is examined.</td>
</tr>
<tr>
<td>Checking the result</td>
<td>None</td>
<td>Answer is given only</td>
<td>Answer is correct</td>
</tr>
<tr>
<td>Maths level of group members</td>
<td>Good-medium</td>
<td>Medium-poor</td>
<td>Poor</td>
</tr>
</tbody>
</table>

Handbook for Teachers in Adult Education
2. Feladat:
leértékelés

A télvégi leértékeléskor 30 %-kal olsóbban tudjuk megvenni a 60 Eurós csizmát.
Mennyiért vehetjük meg?
Mennyit takarítottunk meg?

\[
\begin{align*}
100\% & \rightarrow 60 \\
1\% & \rightarrow 60 : 100 = 0.6 \\
30\% & \rightarrow 0.6 \cdot 30 = 18
\end{align*}
\]

18 eurót takarítottunk meg!

\[
\begin{align*}
-60 & \rightarrow -18 \\
42 & \rightarrow 42
\end{align*}
\]

Tehát 42 euróba került a csizma ára!

Group 1
2. Feladat:
leértékelés

A télvégi leértékeléskor 30 %-kal olcsóbban tudjuk megvenni a 60 Eurós csizmát.
Mennyiért vehetjük meg?
Mennyit takarítottunk meg?

\[
\begin{align*}
100\% & \quad 60 \text{ €} \\
40\% & \quad ? \\
\hline
60 \cdot 0.4 & = 24 \\
+ 42 & = 66 \\
\hline
n\% & = 60 \\
\hline
12 \text{ €} - \text{ért vehetjük meg} \\
18 \text{ €} - \text{ot takarítottunk meg}
\end{align*}
\]
2. Feladat:
leértékelés

A télvégű leértékeléskor 30 %-kal olosóbbban tudjuk megvenni a 60 Eurós csizmát.
Mennyiért vehetjük meg? 10,000,000 euróért
Mennyit takarítottunk meg? 18 euró

\[
\begin{align*}
100\% & \to 60 \text{ euró} \\
1\% & \to 60 \div 100 = 0,6 \\
30\% & \to 0,6 \times 30 \\
18 & \\
+ & 0,0 \\
\hline
18 \text{ euró}
\end{align*}
\]

Group 3
Task 3 Health

1) mathematical situation

Health

A box of vitamin pills that contains 30 pills costs 6 euros. A box that contains 50 costs 8 euros. Which is more economical to buy if we take one pill a day?

2) identifying problems in the situation

Why should I buy the bigger box when the smaller is enough for one month?
I don’t have enough money for the bigger box now.
If I buy the smaller one, I can save 2 euros. I will get payment next month and can buy again
30, my money should not be invested in medicine. This is not good investment, I am afraid
that the pills can go bad with time.

3) planning the problem solving procedure:

One person argues that it is not economical to buy the bigger box because there is no money
left for other things, and tries to convince the others.
They do not try to compare price per pill, they only see that the bigger box costs more and if
something costs more then it is more expensive. There is confusion about the price of one box
and one pill. Finally the teacher has to help.

4) Problem solving procedure:

The discussion in the groups leads to confusion. It is difficult for them to understand the
problem. How can you divide 6 into 30 or 8 into 50? First they proceed in this way, make
calculations and then say that this is the correct answer. They are asked by the teacher to
illustrate their answer by making a drawing, however, they cannot do it. The teacher has to
help to draw the conclusion that price per box and price per pill has to be compared.
However, learners are not convinced by hinting at the solution by writing 6/30>8/50. They
prefer smaller numbers believing smaller numbers mean smaller price. They cannot make a
drawing of the problem and they cannot use a common denominator 30/150>24/150 so they
cannot make a comparison. It is too difficult for them to work with fractions.
Then the teacher suggests that they express euros in euro cents, which helps Groups I and II.
Group III does not accept the suggestion, they do not receive an answer supported by
calculations. By convincing each other and using their intuition, they decide that the larger
box is more economical.
3. Feladat:
    egészségügy

Egy 30 db vitaminkapszulát tartalmazó doboz ára 6 Euró.
Ugyanez a vitamin 50 db-os csomagolásban 8 Euró.
Melyiket gazdaságosabban megvásárolni, ha naponta 1 db-ra van szükségünk?

\[
\begin{align*}
30 : 6 &= 5 \\
50 : 8 &= 6,25 \\
&\approx 6.0 \\
&\approx 6
\end{align*}
\]

Az 50 db-os eink meg jobban.
Mint az 50 db-osnak 1 db-ra 2 százalék lehet végül.

3. Feladat:
    egészségügy

Egy 30 db vitaminkapszulát tartalmazó doboz ára 6 Euró.
Ugyanez a vitamin 50 db-os csomagolásban 8 Euró.
Melyiket gazdaságosabban megvásárolni, ha naponta 1 db-ra van szükségünk?

\[
\begin{align*}
50 : \approx 6,13 &\approx 80 \\
&\approx 80 \\
&\approx 8 \\
30 : 6 &= 5
\end{align*}
\]

50db-osat úgy tevékenyítve többet is, van benne még
menn a hátszerek
Comparison of the work of the three groups

<table>
<thead>
<tr>
<th>Aspects</th>
<th>Group I</th>
<th>Group II</th>
<th>Group III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Applied maths operations</td>
<td>1 box = 50 pills</td>
<td>Euro is expressed in euro cents and the price of 1 pill is calculated.</td>
<td>First they want to make division in the opposite way</td>
</tr>
<tr>
<td></td>
<td>1 box = 30 pills</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>The price of 1 pill is calculated in euro cents.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Drawing conclusion</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Procedure</td>
<td>Putting down everything in writing</td>
<td>Everything is put down in writing step by step</td>
<td>No calculations are made, only a comparative answer is given</td>
</tr>
<tr>
<td>Checking the result</td>
<td>Only answer is given</td>
<td>Only answer is given</td>
<td>-</td>
</tr>
<tr>
<td>Maths level of group members</td>
<td>Good-medium</td>
<td>Medium-poor</td>
<td>Poor</td>
</tr>
</tbody>
</table>

3. Feladat: egészségügy

Egy 30 db vitaminkapszulát tartalmazó doboz ára 6 Euró.
Ugyanez a vitamin 50 db-os csomagolásban 8 Euró.
Melyiket gazdaságosabb megvásárolni, ha naponta 1 db-ra van szükségünk?

\[
\begin{align*}
1 &= 50 \\
1 &= 80 \\
1 &= 100 \text{ cent} \\
6e &= 500 \text{ cent} \\
8e &= 800 \text{ cent}
\end{align*}
\]

\[
\begin{align*}
30 \text{ db-áram c} &= 600 \text{ cent} \\
40 \text{ db-áram} &= 600 \text{ cent} \\
50 \text{ db-áram} &= 800 \text{ cent} \\
10 \text{ db-áram} &= 800 \text{ cent}
\end{align*}
\]

maximális készüléket az összetevő

Group 1
3. Feladat: egészségügy

Egy 30 db vitaminkapszulát tartalmazó doboz ára 6 Euró.
Ugyanez a vitamin 50 db-os csomagolásban 8 Euró.
Melyiket gazdaságosabb megvásárolni, ha naponta 1 db-ra van szükségünk?

30 db a váza 600 = 6000 cent
1 db 100-
50 db a váza 8 • = 800 cent
1 db 800-

A vázasok összegük 8
200 10 ->

A mozdulat hírhedik és leegyedtött
600:30 = 20 Euró
800:50 = 16 Euró

második kisütemény oldalában.

Group 2
3. Feladat:
egészségügy

Egy 30 db vitaminkapszulát tartalmazó doboz ára 6 Euró.
Ugyancsak a vitamin 50 db-os csomagolásban 8 Euró.
Melyiket gazdaságosabb megvásárolni, ha naponta 1 db-ra van szükségünk?

30 db = 6 €
50 db = 8 €

Az 50 db gazdaságosabb, mert az 6 db 6 Euró, ach 5 db 8 Euró. Meg akkor elég, hogy az ötven darab 8 Euró legyen gazdaságosabb.
### Common for Tasks 1-3

#### 5) check the result:

Members of the group tried to convince each other about the correctness of the way they used their knowledge to get the results. Most of the time they took it for granted that the solution to the problem they got as a result of working together was correct, they had no doubts. They never said that what the other is saying is incorrect. Probably this is due to their life situation. They stay together in the same institution and they know that what the other is saying is usually right or wrong. They do not express their doubts because they may need the assistance of the other at any time and they do not want to lose their trust.

Checking the results was done in different ways. They enjoyed drawing the most although they did not find it easy. They used pie charts, number lines and money cards.

#### 6) review the process

Learners got a feeling of learning from each other. They learnt to accept that other people can have good ideas. They had a different attitude to each other after the exercises because they had respect for the ideas coming from the others. They were willing to receive help from others and to give help to others.

When they got stuck with a problem or could not agree on a common way of finding the solution, they asked for the teacher’s help. It was difficult for them to illustrate the problem by making a drawing. They started their drawings many times and often it was just scribbling. They use numbers in their thinking however they cannot do the mathematical operation. As the groups were heterogeneous there was always one person who led the group in the correct direction to solve the problem.

Adults can retrieve existing knowledge and it is easier to apply it to a new problem if they work together:

- **Task 1 (Family budget):** 10%340%=50% and 50% is the half of something, which was familiar to all of them and helped a great deal in working out the solution.
- **Task 2 (Sales):** They have all studied calculation of percentages, some of them even wanted to use the formula (but couldn’t remember it). It was easier to make conclusions by using 1% because they could remember the sequence of the two operations first 1% then 30% i.e. divide by 100 first then multiply by 30.
- **Task 3 (Health):** Comparison can be made after calculating the price of one pill. However, they cannot do the division!

**Review of mathematical problems emerged**

- Comparison of fractions cannot be done even after clarifying the concept of fraction
- When it comes to division it is always the bigger number that is divided by the smaller number, they cannot do the operation in the other way. It is assumed that the result received in this way will be correct too.
- Drawing conclusions in tasks involving proportionality however using 1 unit to calculate more is familiar
- They do not know what to do with zero when multiplying
- Drawing conclusions when calculating percentages and defining the fraction
- Doing mathematical operations is often a problem (digits, decimal point, smaller-bigger)
Teacher’s evaluation

The collaborative learning technique works very well in solving mathematical problems. They need positive strengthening: stress should not be put on pointing out mistakes but they should be encouraged to acquire new knowledge through reinforcing previous knowledge. Work is done the most efficiently if the teacher treats learners as equal partners because learners more easily reveal their weak points.

Adults enjoy playing games and drawing while doing maths, they are very motivated when the tasks can be related to money issues. A lot of side issues emerged through doing mathematics: what is the real price of boots? do medicine go bad in 1 month, why should I buy the bigger box when the smaller is enough for one month and I don’t have much money? etc.

It was surprising to the teacher how difficult they found the task with pills. They can divide a bigger number even by two-digit divider but they cannot divide a one-digit number even by a two-digit divider. Their working with fractions is poor, they cannot even compare which fraction is bigger.

Adults with low literacy and numeracy find it difficult to do complex mathematical tasks. The family budget was a complex task and the learners could not have done it without the help of the teacher. Solving Task 2 required the knowledge of one problem i.e. calculating percentages. The learners had skills in tasks that involved drawing conclusions. They were insecure about doing mathematical operations. They had difficulty in checking the result and often did not even try. Transforming the written task into the language of mathematics was very difficult for them. They don’t understand the problem or they assume that they do but make mistakes in calculations.

Conclusions for the teacher

- Giving tasks that can be solved with one mathematical operation.
- The tasks, if possible, should be illustrated with drawings in order to support learners’ thinking.
- Trying e.g. allowing the use of a calculator to see if they can come to the correct solution if they are not stressed by doing mathematical operations.
Netherlands
MiA Field Project

Learning/coaching/supporting/challenging/facilitating/teaching experiments

The Netherlands - ROCMN
Teacher: Piet van Rheenen

Theme/topic: Discount

General information:

The experiment took place on 16 January 2006 in a classroom of the ROC Midden Nederland, a school for adult and vocational education during a regular lesson in the morning from 9.00 h. till 11.30 h. with a break from 10.15 h. till 10.30 h.

The teacher had 8 adult students:
1. Soraya, woman, 35 years, from Afghanistan
2. Lien, woman, 32 years, from Indonesia
3. Nhung, woman, 25 years, from Vietnam
4. Selvete, woman, 38 years, from Bosnia-Herzegovina
5. Zinab, woman, 41 years, from Morocco
6. Carla, woman, 28 years, from Indonesia
7. Joyeuse, woman, 34 years, from Nigeria
8. Samir, man, 45 years, from Iraq

All students have had formal education in their own countries for at least 5 years with a maximum of 9 years. The women have not learned a profession (housekeeping), the man has qualified as an electronics technician.

All students have been tested to determine their mathematical level. After that they have been following a mathematical course at school at a basic level since September 2005. This can be compared to primary school level for children between 8 and 11 years old.

During the mathematical lessons learning the Dutch language is always an important issue. The meaning of special words and sentences gets specific attention. Every maths-teacher is therefore also a second-language-teacher.

The students work in two groups of four. This grouping is important for learning to cooperate, talking together, learning from each other, teaching each other, so: interaction is critical.

During the whole lesson the method of: Think – Share – Exchange is being used.

- The “Think” part is brief, 1 – 5 minutes and is meant for each student to read, watch and think on his own.
- The “Share” part is longer, 5 – 10 minutes and has the purpose for small groups to tell each other what they have read, seen and thought; the teacher supports this process and coaches the groups.
- The “Exchange” part also takes 5 – 10 minutes and is meant to discuss among the whole group what has been read, seen and thought. The teacher supervises and coaches this process mainly by asking guiding questions and summarising.
**Step 1: Bring the learner in a potential mathematical situation, e.g. sales**

Students always take materials with them to school for their maths lessons. The teachers adjust his topics to these materials. They can come from their placements, which are part of their Dutch as a second language, but also what they see and hear in their daily life. The week before the lesson many brochures, add materials were brought that the students had received through their letter box. Especially in January during the sales period many brochures are delivered. Discount is visualised in different ways in these brochures. “Now for a price of € …: Get 2 Pay for 1: 1 for € 5.95 and 2 for € 10.00: 10%, 20% 25%, 40%, 50% discount: Up to 50% discount: At least 50% discount: Now with 25% more content (paint): etc”. The lesson starts with looking at these sales brochures. The teacher has also brought some along.

**Step 2: Identify (mathematical) problems in the situation**

Each group gets a number of brochures.
Teacher: Why did you take these brochures?
What do you and what don’t you understand?
During the conversation that follows it turns out the students have the following problems:
- Meaning of words and phrases, such as: advertisement, offer, discount, final week, vouchers, up to (40% discount), at least (50% discount).
- The notion percentages. Some students think they know it (50% is € 50 less, 25% is € 25 less), some think they understand it a little bit (50% is half of ..?), some don’t understand at all.

After dealing with the meaning of words and phrases the teacher focuses on the notion of percentages. In one of the brochures it is said: "Final Sales, Everything has to go, The shop has to be 100% empty”
Another brochures says: “50% discount but 100% quality”
Yet another: “100% cotton”
The teachers adds: “You are 100% correct”
So the main question is: “What does 100% mean?”
Step 3: Planning the problem solving procedure

Teacher: A brochure from an optician says: “This week 50% discount on all glasses.”
How much do you have to pay for a pair of glasses of €120?
Get a pen and paper. Do the calculation on paper.
I would like to know how you did it.
During the interaction it turns out that there are three answers: €75, €70, €60.
Teacher: Why do you think it is €70?
How did you calculate this? What do you think?
Can you do it again? We want to hear it.
Can you do it on the blackboard? We want to see it.

It turns out there are three methods shown on the blackboard:

<table>
<thead>
<tr>
<th>Answer €70 or €75</th>
<th>Answer €60</th>
<th>Answer €60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soraya: €120 - €50 = €70</td>
<td>Joyeuse: 120 x 50 = 60 120 - 60 = €60</td>
<td>Nhun 50% is half – half, so half of the price = is half Half of €120 is €60</td>
</tr>
<tr>
<td>Carla: I just think it is €75, I can’t tell you. I can’t remember. Carla probably also calculated €120 - €60, but she has problems with subtracting and difficulty in explaining.</td>
<td>Joyeuse: 120 x 50 = 60 120 - 60 = €60 50% is times 50 and then divided by 100 I have learned to put a stripe through 0. I don’t know why.</td>
<td></td>
</tr>
</tbody>
</table>

There are many questions during the explanation of Joyeuse:
“Why times (X) 50?”
“Why do you do it like that?”
Jayeuse can only say:
“Percentage means dividing through 100. 50% is times (X) 100.”
Confusion reigns.
There is more understanding with the explanation of Nhun.
Soraya reacts with “Ooooh!”. Which means something like: ”Now I understand”
Other reactions:
Samir: “I know 50% is half. That is easier with computer in my head”
Lien: “50 % is not €50. 50% is half. Half of 120 is 60.
Nhun:”The drawing is just like a tomato, half – half, 120 is a whole tomato 100%, so half of 120 is 60, 60 is thus 50%, 60 + 60= 120”
Nhun can also check her answer!

Step 4: Problem solving procedure
From the brochure of the optician: A pair of glasses €180. Now with 50% discount.
Teacher: How expensive are these glasses now? Choose any way you like.
Joyeuse continues to calculate it her way:

$$180 \times 50 = 90$$

$$90 + 90 = 180$$

She is looking for security: this is how she has learned it in the past.

Carla and Lien find Joyeuse’s method difficult. They do it differently:

\[
\begin{align*}
€180 & \quad 50\% \quad €90 \\
€90 & \quad 50\% \quad €90
\end{align*}
\]

\[90 + 90 = 180\]

Selvete can’t find the answer quickly. She calculates trial and error:

\[
\begin{align*}
€180 & \quad 50\% \\
70 + 70 & = 140 \text{ not possible} \\
80 + 80 & = 160 \text{ no} \\
90 + 90 & = 180 \text{ yes}
\end{align*}
\]

For all students checking is important.

Conclusion: checking yourself is better than asking the teacher (Is this correct?)

Conclusion: The drawing of Nhung is helpful to many students.

Now somewhat more difficult.

Advertisement of a bed shop: 25% discount on all!

Teacher: A bed of €240. You get a discount of 25%.

- How much do you have to pay for the bed?
- You now know that 50% is half.
- Can the drawing of Nhung help you?
- Use pen and paper.

Samir is again very quick. He writes down €180. The teacher asks him if he can make a drawing for the others and explain his answer? Samir goes to work after a deep sigh: is this a challenge for him?

On the blackboard we see the following solutions:

<table>
<thead>
<tr>
<th>Joyeuse</th>
<th>Carla, Lien</th>
<th>Soraya, Nhung, Zineb, Carla, Lien, Selvete</th>
<th>Samir</th>
</tr>
</thead>
<tbody>
<tr>
<td>Joyeuse again tries it with her own method:</td>
<td>€240 half-half, 50% is half</td>
<td>Samir has his own drawing:</td>
<td>Hij draws:</td>
</tr>
</tbody>
</table>
| \[
\begin{align*}
240 \times 25 & = \ldots \quad 100
\end{align*}
\] | Hij says: |
| It is getting to difficult for her. She now chooses the methods the other students use. | This is 100% | When he draws a cross (X): This is 25% |
| | this is 50% | |
| | | When he draws an elips: |
| | | This is 75%, so you have to pay: 60+60+60 = 180 |
| | | |
| 240 | 120 | 120 |
| 120, is again half | 60 | 60 | 60 | 60 |
| | €60 | €60 |
| They come as far as: 25% is €60, so you have to pay €60. | | | | |
After Samir’s explanation there is much discussion.

Teacher: What is the discount?  Students: 25%
How much do you have to pay?  Students: what remains, the rest, 60 60 60, 180
What percentage is that?  Students: 25 25 25, 75%
How can you see that in the circle of Nhung?

The students start drawing circles on paper and on the blackboard:

Conclusion: it is the same, but you have to draw two circles,
First a circle with half and then again a circle with half

Conclusion: with Samir’s drawing you can check it immediately:
120+120=240; 60+60+60+60=240; 120+120=240

Everyone, including Joyeuse, wants to work with Samir’s drawing.
the teacher gives a model on paper:
Step 5: Check the result

Another calculation is done with the new model.
Teacher: This bed normally costs €600
Now you get a discount of 25%.
How much do you have to pay for the bed?
How can you check whether it is correct?

<table>
<thead>
<tr>
<th>€600</th>
</tr>
</thead>
<tbody>
<tr>
<td>€300</td>
</tr>
<tr>
<td>€300</td>
</tr>
<tr>
<td>€150</td>
</tr>
<tr>
<td>€150</td>
</tr>
<tr>
<td>€150</td>
</tr>
<tr>
<td>€150</td>
</tr>
<tr>
<td>X</td>
</tr>
</tbody>
</table>

€450

How do you check?
Lien: 300 + 300 = 600 150 + 150 = 300
Soraya has made a mistake: 25% is with her €175
Soraya checks: 175 + 175 = 350, not 300
Conclusion: A mistake is not a problem as long as you check your results.
When you check you can be 100% sure.
You do not have to ask the teacher all the time. You are in control.

Students work on other assignments in a similar manner:
- A tap of €42 with 25% discount
- A children’s pyjama €8.99 (so €9) with 25% discount
Checking still works fine: a mistake was corrected through it.

acional assignment:
A bed €720 with 10% discount.
For this he gets an extended model.
This he can do too..
Then the assignment gets harder:
What is the price with 30% discount?
And now even harder:
What is the price with 35% and with 45%?
This is a real challenge for Samir.

With the exchange:
T: What is 50%? S: €300
T: What is 25%? S: €150
T: What is the discount? S: 25%, €150
T: How much do you have to pay?
S: €450
€600 - €150 = €450
€150 + €150 + €150 = €450
An unexpected situation: 2 students leave for a minute, to the toilet.
When they get back the teacher draws the model on the blackboard:

- T: Selvete en Zineb are back.
  Two students. I put down a 2.
  Why here?
  Talk about it in your group.
  Also write down the other numbers.

  Students talk and puzzle together.
  It is a very lively discussion
  There is much writing and erasing.

  Exchange:
  Selvete and Zineb it is 25%
  All students it is 100%: 8 students
  4 students are one group.
  One group is 50%.
  Other 6 students: 75%
  Checking is easy: 2 + 2, 4 + 4

---

**Step 6: Review the process**

Teacher: What did you learn?
Students:
- What a discount of 25% or 50% means
- How much I have to pay
- Checking
- Working together and talking Dutch
- New words and sentences
- Nhung teaches me, Samir too, we learn from each other.
- The teacher helps but that is not always necessary
The Netherlands
Teacher: Piet van Rheenen

Theme/topic: Budget

General information:

The experiment took place on 13 June 2006 in a classroom of the ROC Midden Nederland, a school for adult and vocational education during a regular lesson in the morning from 9.00 h. till 11.30 h. with a break from 10.15 h. till 10.30 h.

The teacher had 8 adult students:
1. Minh Hanh, woman, 28 years, from Vietnam
2. Rahima, woman, 32 years, from Afghanistan
3. Abeda, woman, 34 years, from Afghanistan
4. Deega, woman, 29 years, from Somalia
5. Abdelhadi, man, 36 years, from Afghanistan
6. Aboobeide, man, 28 years, from Sudan
7. Abdi, man, 34 years, from Nigeria
8. Muanza, man, 35 years, from Liberia

All students had education in their own countries for at least 5 years with a maximum of 9 years. The women had no paid profession (housekeeping), the man had different low-scale professions.

All students have been tested to determine their mathematical level. After that they follow a mathematical course since March 2006 in the school at a basic level, comparing basic school level for children between 8 and 11 years old.

In the mathematical lessons is always learning the Dutch language an important issue. The meaning of special words and sentences gets specific attention. Every math-teacher therefore is also a second-language-teacher.

The students are divided in two groups of four. These groups are important for cooperation, talking together, learning from and with each other, in short: interaction.

During the entire lesson the ‘Thinking – Sharing – Exchanging’ method is applied.

- ‘Thinking’ is short, 1 to 5 minutes and is intended to let every student read, look and think on his/her own.
- ‘Sharing ’ takes a little longer, 5 to 10 minutes and is intended to let small groups tell each other what they read, saw and thought; the teacher guides and coaches.
- ‘Exchanging’ also takes 5 to 10 minutes and is intended to talk with the entire group about what has been read, seen and though; the teacher guides and coaches the process mainly by asking steering questions and summarising.
Step 1: Bring the learner in a potential mathematical situation, e.g. budget

In conversations about dealing with money, the students complain about the increased fixed costs: rent, healthcare insurance, gas, electricity and water, etc. Therefore, they have to be very careful about their other expenses, for groceries for example.

Recommendations from the conversations are:
- how much can you spend in the shops per month and per week
- take that money every week and put it in your wallet
- do not pay with your debit card for groceries
- make a shopping list, do not buy anything else
- teach your children not to whine and certainly not in the shops
- estimate what you have in your shopping cart (round off to whole and half euros): it has to correspond to what you want to spend
- keep an eye on special offers, look at the brochures and only buy what you need.

We have paid a lot of attention to estimated calculations already. The students are quite good at it now. They don’t really master looking at brochures critically yet. Students often say that the special offers are not really what they claim to be. They want to know how come.

We decide to spend a lesson on it. The students will bring brochures for the lesson of 13 June. The teacher will also bring brochures of several well-known supermarkets with the weekly offers.

Step 2: Identify (mathematical) problems in the situation

Every group gets 5 supermarket brochures with the weekly offers.
Teacher: Why did we bring these brochures?
What do you and what do you not understand?

When exchanging ideas the students turn out to have the following problems:
- Meaning of words and sentences, such as ‘when it’s gone it’s gone’, advantage, discount, last week, somewhere else, at the purchase of, etc.

There are hardly any problems with product names, because there are pictures of all the products in the brochures. For Muslims it is not always clear whether there is pork involved or not.

We decide to make a shopping list with much-bought products.
The students make a list:
a whole bread, milk (a litre of yoghurt drink), a kilo of cheese, a kilo of meat (chicken fillet), a litre of coca cola, a kilo of vegetables and a kilo of fruit.
The students start looking for the cheapest offers in the brochures, in pairs.
The teacher draws a table on the board and writes down the products and the pairs.
Who is buying the cheapest?
Step 3: Planning the problem solving procedure

The students are leafing through the brochures looking for the requested products.
Some mark the brochures, other take a piece of paper and write down the products and prices.
The students divide the tasks quite soon. They distribute the brochures and then compare the prices they found.
There are discussion about the prices and the products.
“My child only wants coca cola”
“Cans are always more expensive, are they not?”
“Lettuce is per piece, not per kilo”
“But it lasts two days”
Weights are also a problem: it is often a kilo, sometimes 500 grams, but sometimes also 200 grams, 450 grams, 425 grams, 410 grams or 1.5 kilo.
Another problem: you can buy cola in 1 litre or 1.5 litre bottles, but also in 0.33 litre cans, yoghurt drinks in packs of 1 litre and 1000 ml.

Finally, the students write their prices on the board:

<table>
<thead>
<tr>
<th></th>
<th>Abdelhadi and Aboobeide</th>
<th>Hanh and Rahima</th>
<th>Abeda and Deega</th>
<th>Abdi and Muanza</th>
</tr>
</thead>
<tbody>
<tr>
<td>bread</td>
<td>€ 0.79</td>
<td>€ 0.89 piece</td>
<td>€ 0.79</td>
<td>€ 0.79</td>
</tr>
<tr>
<td>milk</td>
<td>€ 0.79</td>
<td>€ 1.39 2 L</td>
<td>€ 0.69</td>
<td>€ 0.79</td>
</tr>
<tr>
<td>cheese</td>
<td>€ 3.90</td>
<td>€ 3.90 kg</td>
<td>€ 2.99</td>
<td>€ 3.90</td>
</tr>
<tr>
<td>meat</td>
<td>€ 5.49</td>
<td>€ 4.49 kg</td>
<td>€ 5.49</td>
<td>€ 4.49</td>
</tr>
<tr>
<td>cola</td>
<td>€ 2.97</td>
<td>€ 2.97 3 L</td>
<td>€ 2.97 3 L</td>
<td>€ 0.59</td>
</tr>
<tr>
<td>vegetables</td>
<td>€ 0.99</td>
<td>€ 0.99 500 g</td>
<td>€ 0.69</td>
<td>€ 0.49</td>
</tr>
<tr>
<td>fruit</td>
<td>€ 0.69</td>
<td>€ 0.99 500 g</td>
<td>€ 0.69</td>
<td>€ 0.69</td>
</tr>
</tbody>
</table>

Then the students want to know from each other:
“Where did you find that cheap cola for € 0.59?”
“A kilo of cheese for € 2.99, that is not possible. is it?”
Students tend to look for the price and do not convert the price in a standard measure (kilo. litre)
Hanh and Rahima do consequently mention the quantity with the price.
Teacher: “What is the problem with the prices?”
Hanh: “For the others I don’t know if it is kilo or litre”
Abdi: “We have everything for 1 litre or 1 kilo. not the lettuce of € 0.49. We are cheap”
Step 4: Problem solving procedure

Teacher: “How can you know the price per kilo or per litre? Look at the brochure again. Talk about it in your group.”
The students look and talk to each other.
The teacher walks by two groups and asks: “Where do you see the words kilo and litre?”
Sometimes it is clear, sometimes it is not. The students do not look at the very small letters under almost every product that is not sold by kilos or by litres. For example:
next to the cola for € 2.97 for 3 1-litre bottles. It says in very small letters: litre 0.99 The teachers points at it. The students suddenly see the small letters everywhere.

Sometimes they have to calculate:
the piece of cheese for € 2.99 weighs 425 grams and there is no price per kilo in small letters. You get 500 grams of grapes for € 0.69, but no price per kilo.
Teacher: “How do you calculate the price per kilo?”
Most students know that 1 kilo is 1000 grams. They tell each other that the price has to be doubled and of course with a ‘nice’ amount: the € 0.69 grapes per 500 grams cost 2 x € 0.70 per kilo. i.e. € 1.40.
The € 2.99 per 425 grams is approximately more than 2 x € 3. i.e. more than € 6.00. so that is too expensive. Then you don’t have to calculate any further.
The students now want to check their prices.
They take a pen and paper and start working.
After a lot of searching, talking and writing, the lists on the board change:

<table>
<thead>
<tr>
<th></th>
<th>Abdelhadi and Aboobeide</th>
<th>Hanh and Rahima</th>
<th>Abeda and Deega</th>
<th>Abdi and Muanza</th>
</tr>
</thead>
<tbody>
<tr>
<td>bread</td>
<td>€ 0.79</td>
<td>€ 0.89 piece</td>
<td>€ 0.79</td>
<td>€ 0.79</td>
</tr>
<tr>
<td>milk</td>
<td>€ 0.79</td>
<td>€ 0.70 1 L</td>
<td>€ 0.69</td>
<td>€ 0.79</td>
</tr>
<tr>
<td>cheese</td>
<td>€ 3.90</td>
<td>€ 3.90 kg</td>
<td>€ 7.00</td>
<td>€ 3.90</td>
</tr>
<tr>
<td>meat</td>
<td>€ 5.49</td>
<td>€ 4.49 kg</td>
<td>€ 5.49</td>
<td>€ 4.49</td>
</tr>
<tr>
<td>cola</td>
<td>€ 0.99</td>
<td>€ 0.99 1 L</td>
<td>€ 0.99 1 L</td>
<td>€ 0.59</td>
</tr>
<tr>
<td>vegetables</td>
<td>€ 1.98</td>
<td>€ 1.98 kilo</td>
<td>€ 0.69 piece</td>
<td>€ 0.49</td>
</tr>
<tr>
<td>fruit</td>
<td>€ 1.40</td>
<td>€ 1.98 kilo</td>
<td>€ 1.40</td>
<td>€ 1.40</td>
</tr>
</tbody>
</table>
**Step 5: Check the result**

With the new results, the choices are discussed.
Now you know the kilo and litre prices.
Who has the cheapest shopping list?
Abdi and Muanza are the winners. They only have to double the price of the fruit.
The others see that making good choices requires careful looking.
Some had not even seen the cheapest bread and meat.
The yoghurt drinks were not always recognised due to the different commercial names.
The cheapest vegetable was the € 0.49 lettuce, but the students often each lettuce as a side
dish and do not see it as an important vegetable.
Fruit was considered expensive by everyone.
Is there no cheaper fruit than € 1.40 per kilo?
A new search through the brochures resulted in a kilo of bananas for € 0.99. Initially.
everyone had considered it too expensive compared to the grapes for € 0.69 for 500 grams.

**Step 6: Review the process**

Teacher: What have you learned?
Students:
- Look through the brochures: you can make money with it!
- Look at the price per kilo and per litre.
- Check by doubling
- Work together and talk in Dutch
- New words and sentences
Task: Look through the brochures you get next week and look for the cheapest prices for your
shopping.
MiA field project.

The Netherlands
teacher: Henny de Haan

Subject: measuring/weighing in the kitchen.

**General information.**

The good practice lessons took place on 4 May and 11 May 2006 from 2.15pm to 3.30pm. The students are attending a catering assistant training, MBO, level 1 at the ROCMN in Nieuwegein.

This is a normal Dutch school for youths as of 16 years.
The students in this class are following a special process. It is called the integrated process. The group mainly consists of foreign students with a low Dutch language level. In order to send them to the job market as qualified, they are also taking Dutch classes linked to the regular MBO level process for the catering department. Except one student, they have all attended at least 5 years of education. Saleha never went to school in Morocco and takes language lessons in the Netherlands.

The teacher had 11 students.
1. Kyra, an 18-year old Dutch student. Comes from practical education.
2. Khalil, a man, comes from Afghanistan, 35 years old.
3. Mohammed, man from Morocco, 17 years
4. Maryam, woman, from Iran, 40 years
5. Cecil, woman from Congo, 26 years
6. Nathalia, woman, from Brazil, 27 years
7. Zeynel, man from Turkey, went to school a few years in the Netherlands, at a practice-oriented school.
8. Edy, man, from Angola, 21 years
9. Sarah, Dutch 18-year old girl
10. Ugur, man, 19 year old from Turkey
11. Saleha, woman, from Morocco, 26 years

All the students took a math test at the start of the school year. The test showed that the students had certain gaps in their knowledge of maths at primary school level.

The students are at the work tops in the kitchen practice room in groups of 2. Everything is available in the class, pans, measuring jugs, water, gas cookers etc.
The assignment is given in the group.
Then the students work individually.
They write down exactly what they do.
The students carry out the task and look at what others have done, then the results are discussed in the group.
Nathalia is writing down what she wants to do.

Ik kook de kleine doosje die uit 400 gram rijst in. Ik heb gemiddeld 100 gr per persoon nodig voor magnetron per 50 gram heb je 100 ml nodig en per voor 4 personen 800 ml.

Per 100 gr. 150 ml water voor 4 p, 720 ml water nodig.
Step 1. Bring the learner in a potential mathematical situation

The teacher provides the following situation.

You are having guest over for dinner tonight. You are going to make rice for 4 persons. First you are going to buy the rice. You can choose between two different boxes:

1 box costs 60 cents
The other box costs 1.45.

The question is:

- which one do you buy?
- how much rice are you going to use
- how much water do you need for the amount of rice you chose.

The students all set to work decidedly. No one just stands there undecidedly. Except for one student, they all read the back of the box.
step 2. Identify mathematical problems
+  
step 3. Planning the problem solving procedure

The first problem is:

1 pack = 60 cents content 400 grams
1 pack = 1.45 cents content 1000 grams
Which packaging is proportionally cheaper?

No one spontaneously starts calculating which packaging is proportionally cheaper. They just check which one is enough.

Except for one student, they all read the back of the boxes. Everyone says they are going to buy the small packaging. No one spontaneously starts calculating which packaging is proportionally cheaper.

The second problem is:

How much rice do you need per person?

It is striking that the students read very superficially, they think very quickly that they know how much rice they need because the information on the back says something about 100 grams of rice. One student never minds about the information on the back and says:

Khalil does not read.
He says: How many people? OK, four, you say. Yes, one person has a large stomach and the other person has a small stomach. Oh, I’ll make 1 kilo, that is sure to be enough.
Teacher: Why do you take the small box?
Oh, there is enough in there.

Most students choose 400 grams for four persons.
Only Sarah thinks she needs 1 ½ pack. That is enough.

Saleha says: I take a bowl of rice. I add two bowls of water later. That is OK. She never uses a measuring jug but she does not have her own bowls at school. She does not look for a solution.

Zeynel starts reading the back of the box in very much detail. He knows immediately how much rice he needs. 100 grams is enough for 1 person. That is how he interprets the first line of the information on the package under nutritional value.
The third problem is:

100 grams = 180 ml of water
How much water do you need for 400 grams?

Some students do not see this problem. You just take a lot of water and everything will turn out fine. Edy wants to start boiling 2 litres of water for 400 grams of rice. He does weigh.

Edy measures the water in a measuring jug.

Myriam looks at the content of the boxes.
The Dutch always cook with so much precision, she says.
Teacher: So, how much water are you going to take?
Well, at home I take 2 or 3 litres, I just make a lot.
Now I’ll take a litre.
Teacher: Why a liter?
She is not exactly sure.
She takes a pan and starts putting water in it.
Maryam puts water in the pan.

The fourth problem is:
How do you measure a certain amount of water?
4 times 180 is 720,
But how much is that in a measuring jug? What do you call that 720?

Nathalia says: “I am going to use the whole small box. I’ll take 400 grams of rice. 100 grams per person.”
Teacher: Why?
It says so on the box.
Teacher: Can you tell me where?
She points to the 1st remark under ‘cooking advice’. (per 100 grams of rice, 180 ml of water)
She calculates 4 x 180 is 720. See calculation.

Then she starts weighing. She pours water in the measuring cup. She neglects the grade marks and starts pouring water in the balance very carefully. Until she reaches 720 grams.
Teacher: Nathalia, why are you weighing water?
I can’t see very well otherwise, it has to be exactly 720.
Teacher: Why do you use the measuring cup?
To pour the water.

Zeynel first read the box superficially.
He says: “I am going to use 100 grams of rice.
For 1 person I added 180 grams of water.
180 water
For 4 persons? 4 x 180 grams.
Yeah, I’m not so good at maths. It is about 800”

Mohammed and Ugur know exactly what they are doing. They take the measuring cup. They
look for 720 ml and pour in the correct amount of water.

Khalil says: “I buy the small box. It’s cheaper. I’ll take 2 litres of water. That should be
enough.”

Cecil also takes 100 grams for 1 person. She calculates:

\[
\begin{align*}
180 \\
\frac{4 \times}{720}
\end{align*}
\]

But then she has no idea what to do with 720.
She takes the measuring cup and weighs to 720. But what does it stand for??
Cecil measures the correct amount of water, but she cannot name what is in it. 720 ??????

**step 4. Problem solving procedure.**

An overview of the solutions.
The solutions in terms of the amount of rice and water.

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>4 x 180 ml is 720 ml</td>
<td>I’ll just take 2 liters. That is enough.</td>
<td>I’m not really sure. 4 x 180 But 800 is enough.</td>
<td>I’ll take a bowl of rice and two bowls of water.</td>
<td>4 x 180 ml is 720 ml</td>
</tr>
<tr>
<td>180</td>
<td>4x</td>
<td></td>
<td></td>
<td>180</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>180</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>180</td>
</tr>
<tr>
<td>720</td>
<td></td>
<td></td>
<td></td>
<td>720</td>
</tr>
</tbody>
</table>

In the discussion afterwards we look at the different solutions.
The opinions are divided about which is the best solution. It is clear that there are two opinions about boiling rice.
- How do you do it at home?
- How do you do it in a company?
At home everyone finds solutions 2 and 4 a good method. But most of them think it should be done differently in a company. Because otherwise you may be throwing away a lot of rice and that costs a lot of money, is the prevailing opinion. So, let’s learn how we should do it in that situation.

Why do they use 180 ml and not 200 ml, Kyra wonders. 200ml is much easier to calculate.

With this statement we set to work in class.

On the blackboard we draw the following table.

<table>
<thead>
<tr>
<th>1person</th>
<th>2 persons</th>
<th>4 persons</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 grams of rice</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200 ml water</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The students fill it in and it is going fine. In the next class we will continue with this on the basis of the following work sheet.

Rice.
I use 400 grams of rice for 4 persons.
180 ml of water is needed for 100 grams of rice.

<table>
<thead>
<tr>
<th></th>
<th>1 person</th>
<th>2 pers.</th>
<th>4</th>
<th>6</th>
<th>7</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>50 grams</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>100 grams</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>1 person</th>
<th>2 pers.</th>
<th>4</th>
<th>6</th>
<th>7</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>90 ml of water</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>180 ml water</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Calculate
1 person needs 100 grams of rice.
I am cooking for 3.
How much water?

I am cooking for 5.
How much water?

Calculate
1 person needs 50 grams of rice.
I am cooking for 3.
How much water?

I am cooking for 6.
How much water?
The measuring cup

This is a measuring cup.

How much can it contain?

What else can you call 1 litre?

Choose from **decilitre** - **centilitre** - **millilitre**

1 litre
the students fill in 10 decilitre, 100 centilitre and 1000 millilitre.

draw 720 ml
draw 72 cl
draw 7.2 dl

Ugur has no trouble drawing the correct millilitres and decilitres in the jug, but he finds centilitres difficult. When it is pointed out to him what he has filled in before, he knows what to do.
Step 5. Check the result

Because we have seen in practice that measuring with a measuring jug is difficult, we practice in the practical training room with all kinds of different authentic texts about measuring (packaging, recipes) with different kinds of measuring jugs.

The students check the amount in the measuring jug by means of an answer card. It contains a measuring jug with the correct amount drawn on it. The students work together and check the cards for each other.

Most students are doing quite well, some are mixing up centilitres and decilitres. Looking at the example and at the work sheet helps when checking.

Step 6. Review the process

At the end:

What have you learned?
- That you have to use a measuring jug.
- That weighing may be important sometimes.
- That there is 1000ml, 100cl and 10dl in a litre.
- That I am not going to use a measuring jug at home, but that I have to at school.
- That I find weighing very difficult.
- That ml, cl and dl is on everything.

Teacher’s evaluation.

I thought it was fun to work in different steps, but it is quite an investigation. In daily school life there really is no time to prepare and analyse everything so extensively. It is frustrating to see.

It is very effective to let the students try to find out for themselves first. As a teacher I tend to help students too quickly if they have trouble with something. But they know a lot, they do things differently but it also works, and there is a lot I can’t imagine as a teacher.

There weren’t many discussions between them in my lessons. Students in my class find that very difficult.
Spain
Mathematics in Action – analyses of adults learning mathematics in action to improve courses in adult education

Socrates Programme – GRUNTVIG 1
116676– CP – 1-2004-1-DK-GRUNTVIG – G1

Àgora Association
1.1. Family Budget

First:

Who are the adult participants regarding gender, age, ethnicity, how many?
We were 17 participants. Their ages range from 20 to 55 years old. They are from Spain but from different points of the country (Andalucia, Extremadura, Aragón, etc.)

Names of the teachers
There is one teacher in the class: Elisenda Giner

Dates of the experiment and how long did it take?
22nd February 2006

Where did the learning/teaching experiment take place. (in an educational institution, or in a different situation, as part of a course, or ….. etc.)
The Mathematical experiment took place in an educational institution called Escuela La Verneda-Sant Martí

Please describe details:
We were 17 people at class. The academic level is the first one of the Basic compulsory level (three different levels). In the class there are people with different ages, young and adults and a person with a mild mental disability.

1) bring the learner in a potential mathematical situation

The Maths problem was chosen from the Maths Books in the Basic Compulsory level (page 26-28).

Write with only one number the situation (balance) of each person

<table>
<thead>
<tr>
<th></th>
<th>Has</th>
<th>Owes</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ana</td>
<td>10</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>Justo</td>
<td>200</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>Angela</td>
<td>27</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Carmen</td>
<td>4</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>Encarna</td>
<td>50</td>
<td>200</td>
<td></td>
</tr>
<tr>
<td>Abdelkrim</td>
<td>0</td>
<td>27</td>
<td></td>
</tr>
</tbody>
</table>

Ana, Justo and Angela’s balances can be written with natural numbers. However, Carmen, Encarna and Abdelkrim’s balances have to be written with negative numbers. For instance, as long as Ana’s balance is 6, Carmen’s balance is –6.

OPERATIONS WITH NEGATIVE NUMBERS
To add entire numbers with the same sign, we add their absolute value and we maintain the same sign. To add entire numbers but with different sign, we subtract their values and write the sign of the biggest one.

Remember the balance in the situation: Ana 6, Justo 150, Angela 27, Carmen –6, Encarna –150 and Abdelkrim –27

How much money do Ana and Justo have?
How much money do Carmen and Encarna have?
How much money do Justo y Carmen have?
How much money do Ana and Encarna have?
How much money do Ana and Abdelkrim have?

2) Identify problems in the situation

To understand and make the exercise, we have read all together its summary and once finished, the teacher has explained that it is in order to get better comprehension. The teacher asks if everything is clear and the possible doubts are solved using the dialog. The second step is to solve it. We do it without operations. All of them are easy numbers and the problem exemplifies a known situation because all the people at class had to make this kind of operation daily.
From this situation we start to introduce the concept of the negative and positive numbers and the most important, what is its application in the daily life. Once the first part of the exercise is done, we start with its second part. This is most difficult because we have to operate with these numbers and it is something new: we need to know the way to do it in order to find the solution.

3) planning the problem solving procedure:

Concepts: balance, have, owe, positive sign, negative sign
Calculation: add and subtraction

4) Problem solving procedure:

Participants know and understand the concepts. They know how to solve the Maths situation and they learn what a negative number is.
Participants know how to add and subtract, they know as well to multiply, divide with several numbers and decimals as well (although it is not needed for this problem)
Participants don’t know how to operate (add) with positive and negative numbers.

5) check the result:

Participants are able to explain their arguments and calculations:
- they don’t have any problem to explain their arguments to calculate with entire positive numbers in situations of have, owe and balance
- participants don’t have any problem to explain their arguments to operate with entire positive numbers in situations of have, owe and balance when these numbers are simple.
- participants have problems explaining arguments to operate with entire positive numbers in situations of have, owe and balance when these numbers are big ones.
- once learnt the procedure to calculate them, participants don’t have any problem to apply, explain and operate with positive and negative numbers.

6) review the process. What did the learner learn?

What is an entire number
What is a negative number
How operate with negative and positive numbers
First:

**Who are the adult participants regarding gender, age, ethnicity, how many?**
They are 27 participants: 22 women and 5 men. Their ages range from 45 to 75 years old. They are from Spain but from different points of the country (Andalucía, Extremadura, Aragón, etc.)

**Names of the teachers**
Their is one teacher in the class: Milagros Buderos.

**Dates of the experiment and how long did it take?**
7th March 2006

**Where did the learning/teaching experiment take place. (in an educational institution, or in a different situation, as part of a course, or ...... etc.)**
The Mathematical experiment took place in an educational institution called Escuela La Verneda-Sant Martí

<table>
<thead>
<tr>
<th>1) bring the learner in a potential mathematical situation , e.g. sales</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>RECIPE</strong></td>
</tr>
<tr>
<td>The mathematical problem was chosen of the Maths Book called “L’alimentació” (feeding).</td>
</tr>
<tr>
<td><strong>Exercise:</strong></td>
</tr>
<tr>
<td>A person buys a dozen and a half of glass glasses for a party and spends 60€ each dozen. The boxes with the glasses fall down and 10 of the glasses are broken. Which is the price spent in each non-broken glass?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2) Identify problems in the situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>During the dictation of the exercise participants are concentrated in its writing, so once done, each person has to read again in order to understand it: which are the information that we have? Which information the problem asks us?</td>
</tr>
<tr>
<td>Some participants met difficulties to understand that they have to work with unities (of glasses) and no dozens. Why the problem does not say directly 18 glasses instead of a dozen and a half?</td>
</tr>
<tr>
<td>There were some difficulties to understand the summary of the exercise, it was a little bit complicated: “I don’t know how to start” or “I know that I’ve bought 18 glasses but, what I have to do to know the price of each glass?”</td>
</tr>
</tbody>
</table>
3) **Planning the problem solving procedure:**

Participants know that a dozen is 12 units and half a dozen is 6 units, but asked as a Maths problem they have difficulties to write the entire number. They have difficulties to express with numbers what they know in the practice.

4) **Problem solving procedure:**

All the participants, without any exception, know that a dozen and a half are 18 glasses but when you ask for the operation they have used to obtain this number, not all of them know that they use an addition (12+6).

The same situation is in the calculation of half a dozen: all the participants know that half a dozen is 6 units but they don’t know that the operation made was a division. I used this knowledge to explain the concept of “halve”: the half part of 12 is 6. The half part of 50 is 25.

In the present problem participants related the total price of the glasses with 60€, but this quantity corresponds only with the prize of the dozen.

Finally, another difficulty was to operate with different operations (addition, subtraction, multiplication and division all together) and to argument the different steps done. They met difficulties to structure the different steps.

5) **Check the result:**

In this problem they don’t know how many and what kind of operations we have to do to obtain the final calculation, but they understand them perfectly once explained and argued.

6) **Review the process. What did the learner learn?**

In front of a difficult problem like this one, participants’ first impression is: “when you explain us, we understand it perfectly, but it is not possible for us to do alone”. They need a guide, an orientation and this guide I provide with questions and answers.
1.2 Your favourite recipe:

**First:**

**Who are the adult participants regarding gender, age, ethnicity, how many?**
They are 27 participants: 22 women and 5 men. Their ages range from 45 to 75 years old.
They are from Spain but from different points of the country (Andalucía, Extremadura, Aragón, etc.)

**Names of the teachers**
There is one teacher in the class: Milagros Buderos.

**Dates of the experiment and how long did it take?**
22th of February, 2006

**Where did the learning/teaching experiment take place. (in an educational institution, or in a different situation, as part of a course, or …… etc.)**
The Mathematical experiment took place in an educational institution called Escuela La Verneda-Sant Martí

---

1) Bring the learner in a potential mathematical situation

**RECIPE**
The mathematical problem was chosen of the Maths Book called “L'alimentació” (Feeding).

**Exercise:**
In a cooking recipe we can find the next information:
To cook gnocchi we need:
- Potatoes: 1kg.
- Eggs: 3
- Milk: ¼ l.
- Butter: 50g.
- Cheese: 50g.
- Flour: 125g.

2) Identify problems in the situation

This recipe was used to open a dialogue and share experience in cooking and in the importance of knowing what is the meaning, in this concretely case we didn’t know what kind of food it was gnocchi and to how we could calculate the measures in a cooking recipe (gram, milligrams, etc).
This exercise was promoted to look that it appears different measures: kg., g., and ¼ l.
The participants knew what is the difference among these measures: the Kg. was higher than gram, they also knew what ¼ l. was, but they do not know the notation as a fraction.
Every participant prefer to use a spoon to a scale. This was the link to pay attention in next exercises.
In the same exercise, we introduced a new element: > < there were some difficulties to understand this method to say bigger than or smaller than.
Everybody know that the signal to adding is + and the difference between the multiplication that it is represented in x, but it existed the difficulty to understand what it was the meaning of each signal.
3) Planning the problem solving procedure:

At the beginning, we make use of internet in the digital blackboard to look for the meaning of “gnocchi” and some characteristic pictures of gnocchi. The participants have a lot of different resources of memory or memo-technics and to make relation of things. These things are of profit, for example: in dictation to say “with b of Barcelona, etc.

They need simple references to things that they don’t have assimilated before. In the case of bigger than or smaller than we use external terms to identify them: that is as Is the bird beak open or close? That is very useful to remember signals.

Also we can find some participants who say: now I understand but maybe tomorrow we won’t.

4) Problem solving procedure:

One topic that we thought when we were a child it was that first calculation is adding, secondly taking away, to divide, to make fraction,... but that isn’t train to learn to multiply or to divide. Most of them have the problem of memory and they assimilate that they cannot memorize most of the calculations in Mathematics.

Sometimes when they go shopping they solve a maths problem of multiplication using numbers, that it’ll be useful to work on in the maths class.

The teacher often ask to the participants what kind of calculation they have made, one of them say: “adding!”, the correct calculation was taking away and the teacher ask: why did you add and didn’t you taking away this? Suddenly another participant say: it was better to take away because... they often interact among them during the class and motivate other people to participate.

5) Check the result:

In Initial level (certificate) there are different groups of people in knowledge: some of them have assimilated the concepts because they are repeating the same course and they know the answer with the correct justification.

Other people say: “I have added it” and when the teacher ask them, they didn’t know the answer or the reason.

Other don’t know what is the correct calculation but when the teacher explain it in class they finally understand it. And the final group is who have difficulties to understand it.

6) Review the process. What did the learner learn?

The participants learn that every concept a priori is new for them and they will never understand it. All of them have the reference of the first group (who assimilate the maths concepts), they explain to the others that before to understand the explanation in class they had the same difficulties and step by step they started to understand the mechanism and at the end it results easy to calculate it.

This encourages strongly the participants to continue with a bit more of security in themselves. Probably that is one of the topic that is very important work in class: motivate to the participants to have enough security to continue their training and finding different ways to make easy maths adequate them to their characteristics, age, etnia, etc.
First:

Who are the adult participants regarding gender, age, ethnicity, how many?
They are 27 participants: 22 women and 5 men. Their ages range from 45 to 75 years old.
They are from Spain but from different points of the country (Andalucia, Extremadura, Aragón, etc.)

Names of the teachers
There is one teachers in the class: Milagros Buderos.

Dates of the experiment and how long did it take?
22th of February, 2006

Where did the learning/teaching experiment take place. (in an educational institution, or in a different situation, as part of a course, or …… etc.)
The Mathematical experiment took place in an educational institution called Escuela La Verneda-Sant Martí

1) bring the learner in a potential mathematical situation , e.g. sales

RECIPE
The mathematical problem was chosen of the Maths Book called “L’alimentació” (feeding).

Exercice:
In a Danone yoghurt label we can find the following information:
- Protein: 5.2g.
- Fat matter: 5.2g.
- Sugar: 5.3g.
- Calcium: 217 mg.
The net weight: 150g.

2) Identify problems in the situation

We started reading the title.
Once read, they pay attention in a language problem and not in the calculations. This language problem was caused as a consequence of a grammatical question about the word “yogurt” and its plural: in Spanish, we called yogurt so why the plural is different that the singular (yogures instead of yogurtes) So we looked up in a dictionary the original word: we learnt that the origin of the yoghurt is Bulgarian and its original word is yoghourt. The correct translation in Spanish is yogur and not yogurt. The plural is yoghurt.
The participants were right concerning the difference. It is very interesting to talk about everything in the class among participants and teachers.

The next step was to understand the needed calculations. Not all the measures (gram, milligram) are the same so they have to convert them in the same measure to add all the concepts.

A second problem was the decimals and how to operate with them.
3) **planning the problem solving procedure:**

When the participants were writing the problem, they did not know where the comma of the decimals was better to put on (up or down). It was agreed that both places are correct.

It was difficult for the participants to understand the decimals, units, and concepts related.

The 217 milligrams were calculated as grams, they didn’t understand the reason of putting the 0 before the 0.217.

Another confusion was with the decimals is, for example and talking about euros, the difference between 5.3€ and 5.03€. When they read 5.3€ they understand that the amount is five euro with thirty cents and it is difficult to see that it can be five euros with three cent as well. That is, they don’t understand the difference between 5.3€/5.03€.

Something that is surprising is that doing the exercise with real coins there is no any confusion.

4) **Problem solving procedure:**

When we have to work with the quantity 217 mg, they don’t notice the difference between “g” and “mg” and, more important, the meaning of grams and milligrams or their relation with the unit.

When we transform the 217 mg into grams writing the zero before the 217 (0.217) some participants remembered that “the zero in the left is not important, it hasn’t any value”, so it was important to analyse that this idea isn’t true in all the cases. There are some exceptions, like the decimals.

Another difficulty when operating with decimals is the place that the numbers have to be. Some participants don’t take into account that they are operating with decimals and they write and operate as round numbers and they avoid the commas.

To solve that situation, first of all we write the commas (in English dots), later the numbers in the right and finally the numbers in the left.

5) **check the result:**

I have noticed that the first time that the concept of decimal number is explained is difficult to understand. But after some examples, exercises and using concepts and situations of the daily life is easier to understand and use concepts such as unit, decimal, a hundredth, and which one is bigger or smaller.

6) **review the process. What did the learner learn?**

In this problem I learned to write correctly the word “yogur”

I also learned how to use it to check other concepts taught previously.

Sometimes it seems that participants have learned nothing but that is not true. Most of them, especially those who are in the second or third year, use and refer to examples (“but if this exercise is like the another one of the yoghurt…”)

I think that it is necessary that in learning mathematics, participants have to have daily life
references in order to compare, remember and use them. Because although in the kitchen they use spoons or glasses to cook, we can use that as measures. In these recipients there are x grams (it depends on the size). The difficulty is in the use of the language, we have to agree that it is another language and that we have to use it: that is, instead of say in this glass I can put ten spoons say that the spoon is the tenth part of a glass.

Participant learns to think, to compare, to look for similarities in her/his daily life and finally, to use their own strategies. In the daily life nobody teach us how to add or to subtract but everybody knows how to do and we do daily. So in class what we do is to give the mathematical language, that is, we teach what an addition is and what are its different steps and the same with the subtraction, multiplication or division. We establish an order as well: first the addition, subtraction, multiplication and division… however, how is so difficult to learn to divide?

1.3. Health

First:

Who are the adult participants regarding gender, age, ethnicity, how many?
They are 27 participants: 22 women and 5 men. Their ages range from 45 to 75 years old. They are from Spain but from different points of the country (Andalucia, Extremadura, Aragón, etc.)

Names of the teachers
There is one teacher in the class: Milagros Buderos.

Dates of the experiment and how long did it take?
22th of February, 2006

Where did the learning/teaching experiment take place. (in an educational institution, or in a different situation, as part of a course, or …… etc.)
The Mathematical experiment took place in an educational institution called Escuela La Verneda-Sant Martí

1) bring the learner in a potential mathematical situation , e.g. sales

Health
The mathematical problem was chosen of the Maths Book called “L’alimentació” (feeding).

Exercise:
The heart effects, approximately, is 1633 pulses in 23 minutes. ¿How many pulses will effect the heart in 1 minute?

2) Identify problems in the situation

It is very important to the teacher to explain the difference between 23 minutes and we will calculate just one of them. So the teacher presents the problem as easier as possible to clarify any doubt if they have.

They understand the problem but not how to calculate it or what numbers we will use to make the calculation in next step. That is the reason that the teacher just present the problem meanwhile the participants are thinking in different ways to calculate it.

During the explanation appears questions and doubts and it will be solve among all the participants and teachers share as an interactive group with different points of view, knowledge, etc.
3) Planning the problem solving procedure:

Some of them didn’t understand that when we want to share out something we need to divide so if in the exercise we find 23 minutes in 1633 pulses it’ll be necessary to divide this pulses between the minutes to calculate how many pulses will the heart effect. They understand that it’ll be impossible adding or taking away, just only divide!

4) Problem solving procedure:

They identify that we have to divide to know the correct answer. To make it clear the teacher explain carefully the problem and he/she ask to them if they understand every concept and after this it is better to reflect among all the participants what is the best to calculate this.

That is easy when they understand because is just one calculation and they have the learning to make it.

5) Check the result:

When they looked the problem after they understand they suddenly assimilated what is the correct way to calculate: “it will be impossible to multiply because the pulses will increase ant that’s wrong! The logical way is to divide.”

They think about their maths knowledge and all the calculations they know to eliminate and choosing the correct without taking the first one they think in.

6) Review the process. What did the learner learn?

Learn to think using different ways to calculate the concepts we have is one of the most interesting way to make easy the aim of identify the calculation.

They need this security to go on with other news concepts, explanations, exercises, etc. and make sure that they are right and they have taken the right way.

So one of the most clarify to the participants is using daily situations where they can understand that they use this calculations before learning maths: going shopping, in the bank, etc.

This example secure the knowledge because they can make a comparison of their own situations or activities they make everyday.
First:

**Who are the adult participants regarding gender, age, ethnicity, how many?**
They are 27 participants: 22 women and 5 men. Their ages range from 45 to 75 years old. They are from Spain but from different points of the country (Andalucia, Extremadura, Aragón, etc.)

**Names of the teachers**
There is one teacher in the class: Milagros Buderos.

**Dates of the experiment and how long did it take?**
14th March 2006

**Where did the learning/teaching experiment take place. (in an educational institution, or in a different situation, as part of a course, or ….. etc.)**
The Mathematical experiment took place in an educational institution called Escuela La Verneda-Sant Martí

---

1) **bring the learner in a potential mathematical situation**

**Health**
The mathematical problem was chosen of the Maths Book called “L’alimentació” (feeding).

**Exercise:**
A gram of fat contributes with 9 calories. If I consume 2324g of fat, how many calories do they contribute with? If I would like to divide this fat in a week, how many calories I have to eat each day?

---

2) **Identify problems in the situation**

The problem is to establish the differences among grams, days, calories, etc.

Another problem is to divide the quantity among seven days. All of them know that a week corresponds to seven days, but they don’t know what to do with the number seven.

---

3) **planning the problem solving procedure:**

Sometimes the problem is in the summary itself. In this example, the explanation is confusing, it is writing in an abstract way.

The problem talks about division, so the first temptation is to divide 2.324 among 9.

All the participants know that a week has seven days, but they don’t understand that the problem consists in divide the calories in the seven days of the week.

---

4) **Problem solving procedure:**

It is important to analyse the problem very well. There is a multiplication and its result has to be divided. These two operations in only one problem are difficult to understand for people with little mathematic skills. Then, it is important to divide the problem in the smaller parts in order to facilitate its comprehension.

Another important fact is to emphasize which information the problem is given to me. It is important to read, to understand the numbers, to know the operations to make and why I’m doing these operations and not other possible.
5) check the result:

The difficult of the present problem is that there aren’t intuitive answers as some other exercises have. It is important to establish and follow an organized and logical approach. If the result is correct it is due to making the correct operations. They also know how to explain it, sometimes in a personal way.

The problem is done in different steps: first it is necessary to answer the first question, and later on, the second. It is easier. It is important to emphasize that the answer to the first questions give valuable information for solving the second questions. In this way, they can understand that most of the problems include different operations.

When we check the exercise, it is important to do both correct and incorrect operations and to argue why they are correct or not.

6) review the process. What did the learner learn?

It is important that participants argue about the process for obtaining results.

1.4. Evaluation of the experiments

Participants know basic operations but it is difficult for them to apply even if the problems are based in daily life practices. Many of the problems ask for questions that they never have think about.

Sometimes we forget that participants have developed their own logical Mathematics processes. In many occasions, before ending the reading of the summary, most of them know the answer because they have calculated mentally. The mental arithmetic is part of their lives; they use it in the supermarket or in the bank: through a series of additions they obtain a result that probably teachers obtain slowly with complicated Maths calculations.

However, they meet difficulties solving a problem. They are so worried with calculations, because it is so important that the result of these calculations is the correct one. We have to be able to teach that the tools are not the only important ability. It is important that the result of our calculations is the correct one. But it is more important that to know and argue why I am doing this calculation and not another one.

The use of daily examples is basic in the learning of Maths. Solving problems is not so difficult and abstract, we found problems daily and we are able to solve them. The same sensation has to be transmitted in the learning of Maths. Maths is a useful tool.

We have to overcome with preconceptions such as Maths as “difficult” or “boring” or even with the basic rules (addition, subtraction, multiplication and division) is enough.
5. Setup of MiA Teacher Workshops

The MiA Teacher Workshops (MTWs) are developed to facilitate countries to setup professional development for teachers in adult education.

The overall goal is to provide teachers in general and vocational adult education and adult learners with models and examples for how to deal with several real-life situations in which numeracy skills can be further developed such as at the workplace, at home, and in societal life, and to increase adult learners’ motivation by making learning more attractive and relevant.

More detailed the goal of the MTWs is fourfold:

1. Enhance the expertise of mathematics/numeracy teachers in adult education in general
2. Create a common basis for communication between mathematics/numeracy teachers in adult education in European countries
3. Improve the quality of adult mathematics/numeracy education in Europe by developing common starting points in different European countries.
4. Improve the success rate of mathematics/numeracy courses in Europe in general.

In order to achieve these goals the MiA participants created the MiA Teacher Workshops (MTWs) based on the background ideas as described in chapter 3 and on the field experiments in the different countries. The following core elements have been used in the field experiments:

- The theoretical background information about adult learning based on the general starting points of “Learning in practice”, Greeno’s view on “Learning in and for Participation in Work and Society” (Greeno, 1999) and Freire’s theory on “Learning from Experiences” (Freire, 1970).
- Practical background information on the “Six steps” from Van Groenestijn (2002) about how to do real-life mathematics in adult education, and the seven starting points of Freire on “Dialogical learning” (Flecha, 2000).

The setup and structure of the MiA Teacher Workshops were organized during the MiA meeting in Ljubljana in November 2006. The way in which the MTWs can actually be organized depend on local possibilities and facilities. In this chapter the main structure of the MTWs has been showed and how it can be organized. It is not possible to discuss staff issues here because such will differ from country to country.

The organization of the MTWs is based on the following elements:

- Invitation
- Pre-questionnaire
- Ways of organizing and structuring the workshop
- Post questionnaire
- Workshop leader questionnaire
Invitation

Participants of the MiA project created an invitation and application form for the MTWs. This invitation provides information about the MIA project in general and specific information about the workshops. It is supposed to be used in different countries. It can be translated and adapted to local situations. The participating countries used this invitation for their local MTWs during the MiA project.

Questionnaires

There are two questionnaires available for the participants of MTWs: a pre-questionnaire to acquire information about the participants, and a post-questionnaire for evaluation of the workshop.

The pre-questionnaire is based upon the background questionnaire for teachers that was used at the start of the project. It concerns information about the participants’ background education and the number of years of experience they have in adult education. See appendix 5

The post questionnaire concerns evaluation of the workshop. See appendix 6

In an extra questionnaire the workshop leaders are required to reflect on the research questions of the MiA project (see chapter 3). See appendix 7

The materials mentioned above are also available on the MiA website.

The research questions described in chapter 3 can be of help for the teachers to learn more about their learners and their own ways of teaching.

Concerning learning:
1. Why do adults come back to school?
2. What do they want to learn?
3. How do they learn best?

Concerning teaching:
1. Why do we teach adults in adult learning centers?
2. What do we teach?
3. What can be the meaning of an adult learning center for learning in practice, in out-of-school situations?
4. How can we arrange a situation in which the adult learning center can be a center for transfer of learning in a school situation to learning in an out-of-school situation?

The main questions in this are:
1. How can we challenge adults to learn more about mathematics in out-of-school situations?
2. What role can an adult learning center play in supporting and coaching learning mathematics in out-of-school situations?
Ways of organizing and structuring the workshop

The above elements of questionnaires, practical activities and theories and information can be applied in different ways and different order in the MTWs. The possibilities that were field-experimented are described in table (1) below. The ways the MiA countries applied these possibilities are described in table (2).

Table 1: Core elements of the MiA workshop

<table>
<thead>
<tr>
<th>1 Questionnaires</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A pre-questionnaire</td>
<td>B post-questionnaire</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2 Practical activities</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A ice-breaker</td>
<td>B examples of fieldwork in own country</td>
</tr>
<tr>
<td>C practical activity</td>
<td>D examples of fieldwork in other countries</td>
</tr>
<tr>
<td>E design of workshops</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3 Theories and information</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A information about MiA</td>
<td>B 7 principles of dialogical learning</td>
</tr>
<tr>
<td>C six steps</td>
<td>D Greeno’s view on learning for participation in work and society</td>
</tr>
</tbody>
</table>

Table 2: examples of the ways in which the MiA countries organized the MTWs

<table>
<thead>
<tr>
<th>Countries</th>
<th>Examples of possibilities how to organize MTWs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Denmark</td>
<td>1A – 2C – 3 – 1B</td>
</tr>
<tr>
<td>Hungary</td>
<td>3A – 3 – 2D – 2B – 2E – 3C – 2E – 1B</td>
</tr>
<tr>
<td>Lithuania</td>
<td>3A – 3 – 2C – 1B</td>
</tr>
<tr>
<td>Netherlands</td>
<td>3A – 2C – 3C – 1B</td>
</tr>
<tr>
<td>Norway</td>
<td>1A – 3 – 2A – 2E – 1B</td>
</tr>
<tr>
<td>Slovenia</td>
<td>3A – 3B – 2C – 3C – 1B</td>
</tr>
<tr>
<td>Spain</td>
<td>3A – 3 – 2 – 1B</td>
</tr>
</tbody>
</table>
Lithuania
Country: Lithuania  
Title: “Learning Mathematics in Practice”  
Names of leaders: Natalja Kimso, Jolanta Paskeviciene  
Date: April 5th, 2007  
Duration: 7 hours  
Number of groups: 1 group (divided into two while practical exercise)  
Number of participants: 11

Advertising
Invitations were sent to all the Schools and Centres for adults of Vilnius. The teachers could get brief information about the MIA project and on the other hand we got information about the teachers which are going to participate in the seminar on their:
   a. Age  
   b. Gender  
   c. Mathematics teaching experience

Age of teachers:

<table>
<thead>
<tr>
<th></th>
<th>&lt;30</th>
<th>&lt;30-50&gt;</th>
<th>&gt;50</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
<td>5</td>
<td>3</td>
</tr>
</tbody>
</table>

Gender

<table>
<thead>
<tr>
<th></th>
<th>female</th>
<th>male</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Mathematics teaching experience

<table>
<thead>
<tr>
<th></th>
<th>0-5 years</th>
<th>6-15 years</th>
<th>16-30 years</th>
<th>&gt; 30 years</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td></td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Use of and results from pre-questionnaires
We decided to speak about key competences of teachers in adult education and their roles today how to teach adults. Because of the differences in the audience one has experience another does not have.
Structure of the workshops

At the beginning of the seminar the following aspects were presented:
- panel speech on adult teacher’s key competences in the rapidly changing society;
- research on the first teachers questionnaire, presentation on mathematics learning in practice.

The participants were divided into 2 groups and they were given tasks (mathematical situations). Each group could apply 6 steps while solving these tasks. After that there was a presentation of group work. Each group listened to the presentation and asked questions. After the presentation there was a heated discussion. The main topic of the discussion was about 6 steps theory, which is very nice, but each school teaches mathematics according to the educational standards. Teachers affirm that they apply these steps (maybe not all of them), but they don’t emphasize them.

Questionnaire for the workshop leaders

1. Which of the research questions generated more discussion in the group? And what did you find interesting to put in the handbook?

Mostly discussion was held on practical use of the handbook because in Lithuania in formal education we have to teach according Standards of Education approved by Ministry of Science and Education. In - formal Math teaching is not so popular yet. Teachers of Math understand the importance of it but still it is not enough time for teaching practically the subject because of the program and Standards. During the discussion became clear we can teach Math in real life situations through single subject system.

Suggestions to put in the handbook are the following:

- Some theory on Adult Education (ex. M.Knowles) how to teach adults, their psychology, methods, etc.;
- Seven dialogical principles by P.Freire;
- J.G. Greeno statements about adult learning;
- Six steps methodology;
- Practical and theoretical balance;
- Experience from the project partners, examples of exercises;
- How to organize seminars for adults, use of good examples;
- Use of ice breaking methods with examples.

2. In what way do you think the reflection you had in the workshop on Greeno can be useful for teachers’ teaching/coaching practice?

- To use practical examples during teaching/coaching;
- To share students and teachers experience;
- To use authentic situations giving exercises;
- “Go to the kitchen and cook a biscuit” practical use of examples for the teachers, luck of rooms for practical activity.
3. How did you use the seven principles of dialogical learning during the workshop?

1. Participants of the seminar could express their own opinion. It was a real dialogue. There was no fear.
2. Transference. Teachers are given an opportunity to be students and try themselves.
3. Using the dialogue the teacher presents his/her own experience, sharing it with other colleagues and adjusting experience.
4. Tolerance towards other people’s culture and points of view.
5. Use of different kinds of intellectual abilities and their coordination (for example, nice writing, quick reading, observing). While leading the seminar we shared our roles: the first has verbal intellect, second – mathematical, third – physical skills.
6. While presenting group work use of solidarity, help, and assistance.
7. Encouraging and stimulating other group members to work together. Means different motivation aspect, and practical use of knowledge.

4. How did you use the method of six steps during the workshop?
We used the six steps during the workshop as following:
   a. Theory was introduced then we divided participants into two groups (thus using one of the active methods);
   b. Group work;
   c. Presentation of the group work;
   d. Learning was fulfilled with a help of the interactive board
   e. “Learning by doing”;

These 6 steps were accomplished, teachers found it a good idea, but otherwise, teachers didn’t emphasize that it were 6 steps. They really follow these steps did not realize about it.

5. Please write three positive and three negative aspects of the workshop.

<table>
<thead>
<tr>
<th>+</th>
<th>—</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seminar took place during the student’s holidays. Teachers were free from their lessons.</td>
<td>There was lack of time for discussion, everybody wanted to speak.</td>
</tr>
<tr>
<td>Use of modern technologies (Smart Board, Power Point)</td>
<td>Questionnaire has a question about the book. Teachers wanted to see it.</td>
</tr>
<tr>
<td>Open discussion. Every teacher could freely express his/her opinion.</td>
<td>Some of the participants didn’t see the opportunity to use practical examples at their lessons very often.</td>
</tr>
<tr>
<td>We were sure that we create our programme correctly (theory + practice + transference)</td>
<td>We didn’t ask teachers from the secondary schools to come to the seminar.</td>
</tr>
</tbody>
</table>
Norway
Teacher training course in Oslo

Workshop leaders: Svein Kvalø and Christina Berg

Response given to the questionnaire sent out before the course

Where do the course participants teach/work:
 Mostly, in various adult education centres and in The Workers Educational Association.

What kind of educational backgrounds did the teachers have:

3 were trained teachers, 5 were special needs teachers and 4 were teachers with a degree. Only 1 out of 11 participants did not teach adults at secondary school level or basic skills level.

Number of years of experience in adult education

<table>
<thead>
<tr>
<th>Years</th>
<th>1-5 years</th>
<th>6-10</th>
<th>11-20</th>
<th>more</th>
<th>No answer</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3</td>
<td>5</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Experience in teaching maths to adults

<table>
<thead>
<tr>
<th>Years</th>
<th>1-5 years</th>
<th>6-10</th>
<th>11-20</th>
<th>&gt;20 years</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>2</td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

Out of 11 respondents there were 7 who had experience in teaching maths to adults. Only two of the course participants had taken any courses in adult education, internal courses and further education in pedagogic for adults.

Overview of respondents using prepared materials

<table>
<thead>
<tr>
<th>use</th>
<th>Yes</th>
<th>Yes and no</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

Overview of the respondents using personally prepared materials.

<table>
<thead>
<tr>
<th>use</th>
<th>Yes</th>
<th>Yes and no</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>
Comments on what kind of expectations teachers had for this course?

"I have worked with adults for many years, but mostly within ICT. For the first time in my life I will teach adults who have maths difficulties and difficulties in reading and writing at Dynea, a chemical plant outside of Oslo. I hope to get some methodological and academic tips, and some good advices."

"Get some good ideas; may be learn about what is going on at Vox; meet good colleagues."

"Get some new good ideas about how to teach numeracy to adults; tips on good materials; learn about useful websites and software; exchange of experiences with other colleagues in how to teach numeracy to adults."

"Can the ideas we learn about during the course be transferred to Oslo Adult Education Centre?" “Exchange ideas/experiences with others with regards to the practical course with concrete examples and ideas concerning relevant teaching materials.”

"Find something useful for our weak performers at Oslo Adult Education Centre."

"My expectations are to learn about new ways of teaching numeracy to adults. Solving problems by working in groups and having the opportunity to chat with other colleagues in breaks; learn from other course participants. In other words, I’m really looking forward to the course."

Running through the practical part of the course

The course participants were divided into 3 groups of 4 on the basis of the information given in a questionnaire handed out to the participants prior to the course.

There were three stations with different materials:

a) Newspapers with special offers from supermarkets including recipes.
b) Map with a scale. Ruler. Public transport timetable for Oslo.
c) Juice cartons, cream cartons and milk cartons in different sizes. Measuring tape and a folding rule.

Each group was given 30 minutes on each station to prepare a plan for an educational setup on the basis of Mieke van Groenestijn’s 6 points of problem solving.

Each group made an overhead with their educational setup from the first station they attended. After all the groups had been through all the three stations, every group presented their materials. These presentations were discussed with all the others present and the participants were given the opportunity to comment on and give new ideas on the presentations. In addition, every group made notes on paper from the other stations. These notes were collected by Vox at the end of the course. The educational setup summaries will be sent to all the participants by e-mail so these can be used as a small idea bank for participants’ further work in teaching.

The purpose of these practical exercises was to make the participants actively reflect on this particular problem solving methodology. Hopefully, it will give them a sense of ownership of this methodology and get them to implement it in their teaching practice.
6 steps for problem solving - 1

**Material:** A newspaper with special offers published by a supermarket also containing recipes.

<table>
<thead>
<tr>
<th>1. step – Bring the learner in a potential mathematical situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Read and look at the offers in the newspaper.</td>
</tr>
<tr>
<td>What will you have for dinner to day?</td>
</tr>
<tr>
<td>What will you buy?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2. step – Identify problems in the situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>How many will you shop for?</td>
</tr>
<tr>
<td>How much do you need?</td>
</tr>
<tr>
<td>How much does it cost?</td>
</tr>
<tr>
<td>Do you have enough money?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3. step – Plan the problem solving procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Access the amount needed (in relation to the number of people and to the recipe).</td>
</tr>
<tr>
<td>Access the prices (kr per item/package/kg).</td>
</tr>
<tr>
<td>Make a price estimate.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4. step – Follow the procedure and solve the problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Make a shopping list.</td>
</tr>
<tr>
<td>Go to the supermarket.</td>
</tr>
<tr>
<td>Do the shopping.</td>
</tr>
<tr>
<td>Find out how much money you have spent.</td>
</tr>
<tr>
<td>Cook the dinner.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>5. step – Check the result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Did you have enough money to shop with?</td>
</tr>
<tr>
<td>Was there enough food for everybody?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>6. step – Review the process. What did the learner learn?</th>
</tr>
</thead>
<tbody>
<tr>
<td>The process can possibly be evaluated by way of the supervisor observing how the participants solve the tasks.</td>
</tr>
<tr>
<td>In addition the supervisor can interview the participants in relation to the 5th step in the problem solving procedure.</td>
</tr>
</tbody>
</table>
6 steps for problem solving - 2

Material: A newspaper with special offers published by a supermarket also containing recipes.

<table>
<thead>
<tr>
<th>1. step – Bring the learner in a potential mathematical situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>The class is divided into 4 groups all getting the same shopping list.</td>
</tr>
<tr>
<td>For example</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>They will get the same amount of money/credit card.</td>
</tr>
<tr>
<td>The groups will go to different shops to find the different prices of the products.</td>
</tr>
<tr>
<td>Special task: Use a newspaper with special offers and delete the prices per litre and compare the different prices for olive oil.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>2. step – Identify problems in the situation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculate the price in kroner and in percentage.</td>
</tr>
<tr>
<td>How much change will you get back if you pay in cash?</td>
</tr>
<tr>
<td>What will be the price if you pay by credit card?</td>
</tr>
<tr>
<td>Present the results in a table/spreadsheet.</td>
</tr>
<tr>
<td>Special task: Calculate the price per litre.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>3. step – Plan the problem solving procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exercises in adding together prices.</td>
</tr>
<tr>
<td>Exercises in comparing in kroner and percentages.</td>
</tr>
<tr>
<td>How much change will you get back if you pay with a 500 kr note if the merchandises cost 196.50?</td>
</tr>
<tr>
<td>What will be the price if you pay by credit card?</td>
</tr>
<tr>
<td>Make a table.</td>
</tr>
<tr>
<td>Use a spreadsheet.</td>
</tr>
<tr>
<td>Exercises in calculating prices per litre.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4. step – Follow the procedure and solve the problem</th>
</tr>
</thead>
<tbody>
<tr>
<td>Go to the shop and find out how much you have to pay for the items on the shopping list.</td>
</tr>
<tr>
<td>How much change will you get back if you pay with a 500 kr note?</td>
</tr>
<tr>
<td>What will be the price if you pay by credit card?</td>
</tr>
<tr>
<td>Special task: Calculate the price per litre of the different olive oils.</td>
</tr>
</tbody>
</table>
5. step – Check the result
The groups compare prices in kroner and percentages and discuss which store is the cheapest/most expensive.
The groups put their results in a table.
Use a spreadsheet and make a diagram. Some participants show how one can do this.
The participants share with one another how one can calculate the price per litre.

6. step – Review the process. What did the learner learn?
By observing the groups the supervisor can suggest an evaluation as described below:
Some participants will learn how to compare prices in percentages.
Some participants will learn how to use a spreadsheet and create diagrams.
All the participants can learn to compare prices in kroner.
All the participants can learn to find out how much they should get back in change from a 500 kroner note.
Many can learn how to calculate the price per litre.

6 steps for problem solving - 3
Material: Map with a scale. Ruler. Public transport timetable for Oslo.

1. step – Bring the learner in a potential mathematical situation.
The class 6 b from Bygdøy school is going skiing from Skullerud.
There are 20 pupils and three teachers.
They will start skiing at 10.00 am and they have to be back at Bygdøy school at 4.00 pm.
Ticket prices: Flexi ticket 80/160 kr (It can also be a task for learners to find
One way ticket: 11/22 kr find out about the different ticket options).
A "Flexi ticket" comprises 8 rides on any public transportation in Oslo.

2. step – Identify problems in the situation
Find the quickest way of getting to Skullerud by 10.00 am.
Which bus/underground/tram do they have to choose?
When do they have to travel from and to Bygdøy.
Find the cheapest way of travelling for the entire group.
Use a map with a scale to find out the actual distances in reality.

3. step – Plan the problem solving procedure
Suggest different travelling routes by using a map and a public transport timetable for Oslo.
Find the best combination of means of transport to get to Skullerud fastest. Calculate the
travel time and the time spent waiting (minutes/seconds).
Find prices and calculate the total price for the whole group.
Exercises in how to use a scale and a ruler or any other kind of length measuring device.
### 4. step – Follow the procedure and solve the problem

Make a time table for the distance Bygdøy - Skullerud with the help of a map and a public transport timetable.
Departure from/to Bygdøy.
Find the total price for the whole group.

### 5. step – Check the result

If the exercise is done in advance of the trip and the results of the calculations follows, the results can be assessed:

a) did the departure time lead to the desired time of meeting at Skullerud?
b) was the travel time and waiting time calculated correctly?
c) was the total amount of money for the whole group correctly calculated?

### 6. step – Review the process. What did the learner learn?

Observations and discussions during the trip.

---

### 6 steps for problem solving - 4

**Material: Map with a scale, Ruler. Timetable for public transport in Oslo.**

#### 1. step – Bring the learner in a potential mathematical situation

The learners at Sinsen Adult Education Centre are going on a study tour to the National Gallery (Universitetsgata) in Oslo.

The guided tour at the National Gallery starts at 1.00 pm.

#### 2. step – Identify problems in the situation

Find the National Gallery on the map.
Find out how to travel from Sinsen to Universitetsgata.
When must they leave the school if they are to make it for the guided tour in time?

#### 3. step – Plan the problem solving procedure

Practice how to find the streets on the map.
Practice on how to look up different routes on the timetable for public transportation in Oslo and calculate the amount of time it takes to travel on the different routes.
Practice to use the scale to find out the distance on the map.
4. step – Follow the procedure and solve the problem

Use your knowledge to find Universitetsgata on the map.
Use the timetable for public transportation to find out which underground train you have to take.
Find out when they have to leave the school in order to make it to the National Gallery in time for the guided tour that starts at 1.00 pm.
Find the shortest distance between Sinsen and the National Gallery.

5. step – Check the result

Actually go through with a study visit to see if the calculations are correct.

6. step – Review the process. What did the learner learn?

On the next study visit the learners can plan their trip on their own. The teacher can observe how the learners handle this task.

6 steps for problem solving - 5

Material: Juice cartons, cream and milk cartons in different sizes, measuring tape and a folding rule.

1. step – Bring the learner in a potential mathematical situation

Find out by measuring the different sides of a litre cartons if the content equals to 1 (cubic)decimetre which is equal to 1 litre.
How many grams of the different nutritional content is there in a glass of orange juice (2 dl)?
How many glasses of orange juice would you have to drink to get the same amount of grams of proteins as there is in a glass of low fat milk?

2. step – Identify problems in the situation

Conversion of units. 1 litre = 10 dl. 1 litre = 1 dm³.
Measuring of length with a ruler.
Calculation of volume.

3. step – Plan the problem solving procedure

Exercises on converting from litres to decilitres.
Exercises on converting from litres to cubic decilitres.
Exercises on how to calculate the volume of a milk carton (prism).
Exercises on interpretation of the constituents given on the food packages.
4. step – Follow the procedure and solve the problem

Measure the side of the milk cartons and calculate the volume in cubic decimetres. Read through the constituents given on a juice carton and find out the number of grams of protein there are in one glass of juice. Do the same with low fat milk. Find out how many glasses of milk you have to drink to get the same amount of protein as it is in one glass of low fat milk.

5. step – Check the result

The participants compare their calculations and tell each other how to convert cubic decimetre to litre and how to calculate the volume of a milk carton. The participants discuss how they found out about the amount of protein there is in juice and low fat milk.

6. step – Review the process. What did the learner learn?

"I can do” methodology can be applied.

6 steps for problem solving - 6

Material: Juice cartons, cream and milk cartons in different sizes, measuring tape and a folding rule.

1. step – Bring the learner in a potential mathematical situation

The family’s consumption of healthy beverages.

2. step – Identify problems in the situation


3. step – Plan the problem solving procedure

Exercises in measuring of length (cm, m). Exercises in calculating simple volumes. Training in handling mathematical concepts. Exercises in reading and interpreting information on different packages.
4. step – Follow the procedure and solve the problem

Find out how much milk/juice the family needs each week and if there is enough space in the refrigerator for the weekly grocery shopping or if they must they shop more often. We would like to buy 5 litres of milk. There is only 3 one litre cartons left in the store. How many other cartons with a different volume can we buy? Comparing of prices of milk in different kinds of packages. Comparison of fat content in different types of milk.

5. step – Check the result

The participants can share the results they have compiled and discuss among them selves the differences and the similarities in how the solved the different tasks.

6. step – Review the process. What did the learner learn?

All of them have hopefully learnt how to measure length in cm and m and about concepts such as largest/smallest etc. Some of them have learned how one interprets merchandise information on milk and juice packages.
Evaluation of the course

The post-questionnaire was handed out and filled in by the participants after the course.

The assessment of the prior information sent before the course is as given in the table.

<table>
<thead>
<tr>
<th></th>
<th>Very good</th>
<th>good</th>
<th>Neither/nor</th>
<th>bad</th>
<th>Very bad</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The table below shows that most of the participants experienced that what they had written in their questionnaire prior to the course had been taken into consideration.

<table>
<thead>
<tr>
<th></th>
<th>correct</th>
<th>Partly correct</th>
<th>Neither/nor</th>
<th>Partly incorrect</th>
<th>Incorrect</th>
<th>No response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5</td>
<td>2</td>
<td>1</td>
<td></td>
<td></td>
<td>4</td>
</tr>
</tbody>
</table>

The table below shows that the participants were satisfied with the teacher training course.

<table>
<thead>
<tr>
<th></th>
<th>correct</th>
<th>Partly correct</th>
<th>Neither/nor</th>
<th>Partly incorrect</th>
<th>Incorrect</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>9</td>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The overall impression is that the participants were satisfied with the alternating use of theory and practice.

<table>
<thead>
<tr>
<th></th>
<th>Very good</th>
<th>good</th>
<th>Neither/nor</th>
<th>bad</th>
<th>Very bad</th>
<th>No response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>6</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

All 12 participants were very satisfied with regard to freedom of expression during the course.

As the table below shows, the course participants were quite satisfied with the material handed out during the course.

<table>
<thead>
<tr>
<th></th>
<th>Very good</th>
<th>good</th>
<th>Neither/nor</th>
<th>bad</th>
<th>Very bad</th>
<th>No response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>6</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

The course participants liked in general the 6 steps for problem solving.

<table>
<thead>
<tr>
<th></th>
<th>Very good</th>
<th>good</th>
<th>Neither/nor</th>
<th>bad</th>
<th>Very bad</th>
<th>No response</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4</td>
<td>5</td>
<td>2</td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>
The majority of the course participants found that the time used on the course was effective.

<table>
<thead>
<tr>
<th>correct</th>
<th>Partly correct</th>
<th>Neither/nor</th>
<th>Partly incorrect</th>
<th>Incorrect</th>
<th>No response</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>3</td>
<td>1</td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

Some comments from the course participants on what was the most useful part of the course for their work?

“The discussions in the group”, “Contact with new colleagues and exchanging ideas and maths exercises”, Meet new colleagues and learn about the method for problem solving”, “Practice/useful – meet others – exchange experiences and ideas”, “The discussions in the group. Got valuable tips and advice”, “The exchange of ideas with others”, “An inspiration to make tasks in such a way that the learners can use their everyday knowledge and skills”, “Some practical tasks”, “Ideas on how to structure maths lessons, ideas to practical tasks”.

Some of the course participants’ suggestions for improvements of the course.
“More time set aside for each task”; “not enough time to work with each of the three practical tasks”; “two practical tasks instead of three”; “more about the difference between everyday mathematics/numeracy in this context compared to the maths we do for adults at secondary school level”

The table below shows that 10 out of 12 participants are positive to applying the 6 steps for problem solving in their own teaching.

<table>
<thead>
<tr>
<th>yes</th>
<th>no</th>
<th>No response</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

From the participants 9 persons think that the 6 steps can be applied in general to adult education.

<table>
<thead>
<tr>
<th>yes</th>
<th>no</th>
<th>No response</th>
</tr>
</thead>
<tbody>
<tr>
<td>9</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

As many as 10 of the course participants would like to take other courses in this kind of subject.

<table>
<thead>
<tr>
<th>yes</th>
<th>no</th>
<th>No response</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

All the participants who answered would recommend this course to colleagues.

<table>
<thead>
<tr>
<th>yes</th>
<th>no</th>
<th>No response</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>
Overall we are very satisfied with this evaluation. We did not put too much effort in promoting this course. We sent out a few e-mails and the course was advertised as a free course on Vox’s website. We are quite confident that we can successfully run several courses like this one in the future.

Finally our impression is that the ambience was exceptionally good during the session.

The programme:

**Time:** March 2, 2007, 11.00 – 15.15 o’clock

**Place:** Vox, auditorium

The course is for teachers who teach basic numeracy for adults at the workplace and at adult education centres.

The course will focus on a specific methodology for problem solving in a mathematical context connected to real life situations in everyday life.

A booklet containing the theoretical ideas of the course will be handed out.

A free lunch will be served.

<table>
<thead>
<tr>
<th>Time</th>
<th>Content</th>
<th>Who</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.30</td>
<td>Registration and coffee.</td>
<td></td>
</tr>
<tr>
<td>11.00</td>
<td>Welcome. A short presentation of the new framework for numeracy for adults. Recommendations and the theoretical background for the course.</td>
<td>Svein Kvalø</td>
</tr>
<tr>
<td>11.25</td>
<td>A practical example of the problem solving methodology</td>
<td>Christina Berg</td>
</tr>
<tr>
<td>11.35</td>
<td>Coffee break.</td>
<td></td>
</tr>
<tr>
<td>11.45</td>
<td>Practical use of the problem solving methodology. <strong>The first task</strong> (work in groups)</td>
<td>All participants, Svein Kvalø, Christina Berg, Våril Bendiksen</td>
</tr>
<tr>
<td>12.15</td>
<td>Lunch.</td>
<td></td>
</tr>
<tr>
<td>12.45</td>
<td><strong>The second task</strong> (work in groups)</td>
<td>All participants</td>
</tr>
<tr>
<td>13.15</td>
<td><strong>The third task</strong> (work in groups)</td>
<td>All participants</td>
</tr>
<tr>
<td>13.45</td>
<td>Break. Fruit.</td>
<td></td>
</tr>
<tr>
<td>14.00</td>
<td>Short presentation of the work done in the groups. Discussions.</td>
<td>Some participants</td>
</tr>
<tr>
<td>15.00-15.15</td>
<td>Conclusion. Evaluation. Handing out course certificates.</td>
<td>Svein Kvalø</td>
</tr>
</tbody>
</table>
How do adults learn and what is special for adults compared to children?

In practice adults have multiple tasks to fulfil. They have families, are parents, neighbours, citizens, customers, consumers, employers, employees, patients etc. From Freire’s pedagogy and studies about learning in practice we can derive seven general starting points for the learning of adults in Adults Basic Education, ABE:

1. **Adults are free to learn.** They learn because they need or want to be better informed, want to improve specific skills or to acquire more specific knowledge. They are not obliged to learn as they were in primary and secondary school.

2. **Adult learners are equal partners in learning situations.** Adult learners have many real life experiences which may affect their own learning. If teachers and learners are aware of this, there can be a sharing of real life experiences as equal partners in the learning process, which may enrich the learning situation.

3. **Adults’ own experiences are the basis for learning.** Learning starts when adults are aware of what they know and what they want to know. New knowledge should be built on prior knowledge.

4. **Authentic materials should be used as instruction materials in school learning situations.** Following the previous starting point, instructional materials used in learning situations should be authentic or present reality by means of photos, schemes, etc. in order to give meaning to the learning in school and create the link between school knowledge and real life knowledge.

5. **Learning takes place by interaction and reflection.** By discussing new learning topics related to prior knowledge, adults become aware of and gain insight into their own knowledge and skills. Verbalizing thoughts and mathematical procedures in communication with others may also improve people’s mathematical reasoning and communication skills, which are, in turn, preconditions for being able to manage everyday life situation and for cooperative learning situation.

6. **Learning in adult education aims to lead to functional knowledge and skills.** Based on studies about learning in practice, learning in adult education should focus on learning-for-doing rather than learning-for-knowing. Learning-for-doing leads to learning-for-knowing, i.e. functional knowledge.

7. **Adults direct their own learning by constructing and reconstructing, organizing and re-organizing their own knowledge, procedures and skills.** This can be encouraged in learning situations by helping adults to create their own mathematical procedures based on their own insight and skills when solving mathematical problems.
From an andragogical and constructive point of view teachers cannot transmit knowledge; they are just facilitators for learning processes.

AGORA, an Adult Education centre in Barcelona applies ”Dialogical Learning” as a teaching methodology which is inspired by the principles you have been introduced to. Check out: http://www.statvoks.no/mia/lastned/dialogical_learning.pdf

**RME – Realistic Mathematics Education**

RME has five learning principles:

1. Learning is a constructive activity.
2. Learning moves through various levels of abstraction with the provision of models, schemes and symbols.
3. Learning takes place by reflection.
4. Learning is a social activity.
5. Learning mathematics leads to a structured and interwoven entity of knowledge and skills.

In RME, it is important to consider the adults’ practical experiences as a basis when they solve mathematical tasks: Adults must be allowed to try their informal problem solving strategies. This will give the teacher insight into how adult learners think. This creates a good basis for good communication between teacher and learner which again is a good starting point for a profitable learning outcome for the learner. The aim of the process is to guide the learner away from the informal problem solving strategies to formal solving strategies.

![Progressive Schematizing](image)
Greeno’s principles (Learning in and for Participation in Work and Society):

- Learning is fundamental to and a natural part of human activity.
- Learning in a classroom setting is artificial. It is a non-natural situation. Adults learn best in actual real-life situations.
- Learning, motivation and activity are not separable. If adults don’t see the need for learning (e.g. a particular subject) then they may not be motivated to learn.
- Adults don’t learn just to do but to become.
- Learning in practice is based on shared knowledge. Learning in school often focuses on individual knowledge.
- Learning is often situation-based and situation-bound.

Mieke van Groenestijn’s principles for problem solving in combination with the practical solving of mathematical problems in real life situations.

The actual problem solving process shows the “what”-process. To analyze this process with learners the three process steps plan-do-review can be discussed by posing three questions:

What are you going to do? (plan)
What are you doing? (do)
What did you do? (review)

These three questions are questions that teacher and learner have to focus on when they solve a maths task.

The actual process for problem solving that adults go through can be described in six steps (the “what- process”).

1 Bring the learner in a potential mathematical situation.
2 Identify problems in the situation.
3 Plan the problem solving procedure.
4 Solve the problem.
5 Check the result.
6 Review the process. What did the learner learn?

During the practical part of the course we want you to reflect especially on Greeno’s principles and on Van Groenestijn’s process for problem solving of mathematical problems in a real life situation. 10

10 The content of the three first pages of this booklet has been derived from “A Gateway to Numeracy” published by Mieke van Groenestijn (2002).
Recommendation to teachers who teach at the work place.

It is important that there is good communication between the workplace and the teacher who will teach the course. It’s important that the enterprise has a clear understanding of what kind of numeracy skills they would like their workers to have. If this isn’t clear, it’s important that representatives from the enterprise and the teacher meet and discuss what the aim of the course will be. E.g. just to improve relevant numeracy skills at that particular workplace, or prepare the workers for a specific vocational trade.

For the teacher to come to an understanding of the circumstances, and what kind of tasks the employees have at the workplace, it is important for the teacher to visit the enterprise and learn about the tasks the workers have. When the teacher knows what kind of tasks the workers have, it becomes easier for him/her to prepare appropriate teaching materials which are adapted to the workers’ daily tasks. By doing it in this way, it can further motivate the workers to participate since they will find the material useful.

Demands on the teacher

The teacher must be flexible. She/he must set aside time to visit and speak to the employers and the workers at the workplace. The teachers have to be prepared to teach at awkward hours. E.g. from 7.00 am to 9.00 am.

What is special about mathematics?

There are many adults who feel that they don’t have a handle on mathematics. The reason for this is often bad experience in maths at school. This has to be taken into consideration by the teacher when adults return to maths lessons. One can achieve this by starting off with simple exercises which can build their self confidence in handling mathematics. It can be wise to spend some time building the participants’ feeling of success in solving relevant maths tasks.

Since many adults have not put maths into practice for many years, they have often lost skills in doing maths. This group of learners needs to work on maths tasks they can master. As a rule it doesn’t take much for this group to get back to the level they were at once before.

Many adults think of mathematics in terms of algebra and solving equations. It’s therefore important to stress that algebra and equations are not a part of basic numeracy for adults.
Slovenia
Mathematics in Action

Report on the pilot teacher training seminar in Slovenia

Country: Slovenia
Trainers and authors of the report: Andreja Jelen Mernik, Saša Silovšek
Translation: Estera Možina
Completed in Celje, Slovenia, April 2007
Content

1. Introduction to the workshop in Slovenia

   Where and when
   Participants
   Invitation

2. About the workshop

   Introduction to the EU projects
   Introduction to the MiA project
   Review of good practices – exchange of the experiences
     Spanish case
     Danish case

   Workshop
     Shock therapy
     Practical exercise
     Problem solving
     Identifying the steps of problem solving
     Introduction to the 6 steps and
     Comparison with own conclusions
     Discussion

3. Evaluation of the workshop
1. Introduction to the workshop in Slovenia

1.1 Where and when

The workshop, which lasted 3 hours, was organised for teachers in adult education employed in the private educational organisation (the title of the organisation is INVEL) in the town Velenje, situated in the NE of Slovenia. The organisation is dealing mainly with adult education and training and business counselling for different target groups in the region e.g. unemployed, pupils and students, organisations and individuals in business sector. The workshop was carried out on 5th of April 2007 in the premises of the organisation.

1.2 Participants

All the participants were woman teachers; with up to 10 years of teaching experiences, with the university degree, all of them completed the training for adult literacy teachers at Slovenian Institute for Adult Education for teaching different target groups of adults and have some experiences in teaching in those programmes. Altogether there were 7 participants on the workshop.

1.3 Invitation

The participants received invitation with the programme, duration, time and place of the workshop as agreed on the last seminar in September Ljubljana. The evaluation questionnaire was not sent in advance in purpose, it was established that there was enough information about the participants to start the workshop and that the questionnaire will be distributed in the end.

Additional to the common invitation:

INVITATION TO THE WORKSHOP

MATEMATICS IN ACTION

To be held in the premises of the organisation INVEL, in Velenje, on April 5th, 2007, starting at 12.00.

Programme:
1. Introduction to the EU projects
2. Introduction of MiA project
3. Examples of good practice
4. Six steps model, workshop
5. Discussion
2. About the workshop

Workshop was carried out following the programme set up in advance:

2.1. Introduction to the EU projects

- Introduction to different EU projects
- Cooperation
- Financing
- Adult education projects

2.2. Introduction to the MiA project

- Partners
- Aims
- Stage of the project and work completed

2.3. Review of good practices – exchange of experiences

2.3.1. Spanish case

Characteristics of dialogical learning:
- Learning together.
- Learning from each other with materials produced by the learners.
- Independent learning.
- Teacher organises learning and does not just transmitting knowledge.

The results of our small experiment of learning statistics were introduced shortly to them. As reported the experiment was carried out in two groups of pupils, in the first with classical approach, teacher explains – pupils listen and do the tasks. In the second group the teacher distributed the tasks and pupils search for the solution themselves and learn form each other. The main finding of the experiments was that retention of the knowledge was approximately the same immediately after the experiment but after 1 month the retention of knowledge was better in the second group in which pupils learn from each other. So the conclusion was, taking into account the small scale of the experiment respectively, that the learning was more efficient in the second group.
The 7 principles of dialogical learning enhance:

1. adults learn free will (adults are not forced to learn, it is based on their own decision and need)
2. learning is linked with concrete situation (learning is based on learners needs)
3. real life materials and data are used in the process of learning
4. learning situations are socially and culturally relevant to learners
5. learning is based on co-operation among learners and others
6. basic steps in learning are: observe – simulate – experience -do.
7. learners create their own principles and guidelines how to use knowledge and skills acquired in new situations in their life.

Trainers’ comments:
Interesting discussion among the participants started after this section. What was said was interesting to the participants of the seminar, they were able to recognise those principles in their practice (they take into account social and cultural backgrounds of the learners; they found similarities with their work with adult learners. They emphasize the importance to take into account learners needs in adult education. All participants recognised that they use the 7 principles in their practice, but most of them do that intuitively, they are not aware of that. The introduction of 7 principles was very positively accepted by participants as useful for their teaching practice.

2.3.2. The Danish case

Participants were shortly acquainted with the learning math at the workplace in Denmark. Basic information was given about the programmes, when and where such programmes are carried out, duration of the programmes, financing, progression routes, etc. It was also emphasised that the situation cannot be compared with present Slovenian situation.

Trainers’ comments:
After the presentation lively discussion started and several questions were asked. The participants were comparing the Slovenian programme of workplace literacy with the Danish math classes on the workplace. Basic conclusion was that the Danish experiences are different, Slovenian employers are not interested supporting the education of least educated employees. Also, the state support is not sufficient. At the moment only the Slovenian Ministry of Education is supporting the literacy at workplace as a part of basic adult education and development of literacy skills.
2.4. The workshop

This was the main part of the seminar and most of the time was planned for this part. Comments and discussions were very interesting, most of them were written down, but not recorded.

2.4.1. Shock therapy

Goals of the activity: to give the participants opportunity to experience how difficult is to do or to learn things which make no sense or are not relevant to them.

The task asked participants to transcript carefully the text given to them on a sheet of paper (the purpose was to perform the role of adult learners and experience their feelings).

The trainer shall explain in the introduction that several skills could be practiced with such an activity (individual trainer can always present and justify his own goals of the activity, depending on the profession of the participants): observing, speed, handwriting, patience, concentration. Participants see no sense in such an activity. In similar situation are adult learners most of the time, when teachers starts with general introduction and explains general principles or else. Similar way math is taught in schools to adults, they consider such knowledge not useful, and learning is not long term. Teacher shall start from everyday relevant situation for learners (not from situation which teachers consider relevant for adult learners).
Trainers’ comments:
Participants realised that that was just an exercise, but started to transcript anyway. What they did not know was the purpose of the activity. Because they did not see any sense in the activity they asked question such as how accurate must the transcription and similar. After the activity the discussion started about their feelings and they commented that the activity was useless and made no sense, despite the fact that they were told that several skills could be enhanced. The result of the exercise was very good; the discussion relaxed the participants and very soon they recognise emphatically that adult learners must feel the same when given irrelevant topics. The goal of the task was accomplished.

2.4.2. Practical exercise

In introduction to the workshop it was emphasized that:
- the group will work on practical example,
- the goal is not to find a result but to observe the process of thinking and searching for a solution,
- this process has always certain steps and if we became aware of them we can get a model for learning,
- participants shall observe each individual process and try to find their own model.

2.4.3. Problem solving and observation of the process

Task was given to the participants as follows: The classroom will be decorated by a new floor, the floor need to be rimmed with wooden board. The task in to find out what is the length of the board?

Trainers’ comments:

The group spitted into two groups and started to solve the case:
- first the groups found out that they need the information on the size of the room and they need to measure the width and the length, before calculating,
- they established they needed a meter,
- some participants made a sketch of the room,
- they realised that they do not need a formula but they can simply add all of the measures of the room,
- they realised also that the board is not needed at the doors,
- they found out that one of the walls is not straight, they will have take into account,
- they estimated the size of the room with steps,
- when the data were collected, all of the measures are simply added.

They stopped at this point and the trainers break with the question; do you think you were accurate enough that you can go to the shop and buy the wooden board?
The answer was straight; they would need more accurate measures. They suggested that the result can be checked out with the tailor’s meter which can be flexible. The task was done for them, but the trainers asked the question: Have we learned anything from doing this activity? The answer was, that would know how to measure any kind of room.

2.4.4. Identifying the steps of problem solving

The steps identified by them solely were written on the blackboard:

2.4.5. INTRODUCTION OF THE 6 STEPS AND COMPARISON WITH OWN CONCLUSIONS

The method of 6 steps of problem solving has been introduced to the group at this point.

Teacher’s comments:
The comparison was made afterwards between the theory and their experiment. The difference was established and described. The group then explained the difference and identified the reasons as follows:
- participants combined the first two steps because they considered the task easy, and represent no mathematical problem to them,
- if the task would be difficult they would have to be more careful and the steps would be more visible.
2.4.6. DISCUSSION

At this point the trainers explained the group what was their main task:
1. to pilot the workshop with the teachers teaching adults,
2. to analyse in discussion with them the method and the approach in instruction of teachers
   (doubts between telling about the method in advance or give the opportunity to try out by themselves),
3. to record responses and reactions of teachers,
4. to evaluate the pilot approach,
5. to discuss with the group what they need, what is useless, new suggestions…).

In short to improve the teacher’s workshop.

Trainer’s comments:
Soon an interesting conversation started, first about the workshop itself and then about their concrete problems and learning situations with adult learners. The opinions of the participants are collected in the evaluation questionnaire. Some of their suggestions are that the handbook shall contain:
- examples of successful workshops completed together with all of the activities (with the learning tasks and materials),
- list of ideas for workshops with adults organised by topic,
- examples shall be very concrete and relevant, concrete examples are needed for this target group,
- not a lot of theory, the theory shall be supported by practical examples.

In the end the evaluation questionnaire was completed by all of the participants.
3. The evaluation of the workshop

The answers of the teachers on the evaluation questionnaire:

1. The information about the workshop received beforehand was:
   A – excellent  B - good  C - reasonable  D - satisfactory  E- unsatisfactory
   1  5

2. The workshop met my expectations.
   A - true  B - partly true  C - no opinion  D - partly not true  E - not true
   3  4

3. The balance between theory and practical work was:
   A – excellent  B - good  C - reasonable  D - satisfactory  E- unsatisfactory
   6  1

4. I was able to express my opinions freely.
   A – true  B - partly true  C - no opinion  D - partly not true  E - not true
   7

5. The materials I received during the workshop were:
   A – excellent  B - good  C - reasonable  D - satisfactory  E- unsatisfactory
   4  3

6. The methods the workshop leader used were:
   A – excellent  B - good  C - reasonable  D - satisfactory  E- unsatisfactory
   7

7. Time was used effectively.
   A - true  B - partly true  C - no opinion  D - partly not true  E - not true
   7

8. What was the most interesting/useful for your work?
   – usefulness in the programmes carried out by the teachers,
   – relevance of the problems presented and usefulness for our own context and practice,
   – identifying the problem and solving the problem (method of the 6 steps),
   – active participation.

9. Do you have any suggestions for improvement?
   No suggestions, the approach is good.
10. Are you planning to use the six steps or seven principles in your teaching practice?
   A – yes (7 answers)
   Please, explain:
   - in the adult literacy programmes (7),
   - I will use the method similar way with the participants (2).
   B – no.
   Please, explain (0 answers):

11. Do you think these methods and principles are useful also in education in general?
   A – yes (7 answers).
   Please, explain:
   - method enable that the participants research and discover by themselves (7),
   - the retention of knowledge is better (3),
   B – no (0 answers).
   Please, explain:

12. Are you interested in further training related to these themes?
   A - yes  B – no
   7

13. Are you interested in further training related to these themes?
   A - yes  B – no
   7

14. Would you recommend this workshop to your colleagues?
   A - yes  B – no
   7

15. Please, explain in what way you expect the MiA teacher’s handbook would be the most useful for your work?
   - will improve my teaching and explanation (1),
   - shall contain concrete examples and concrete learning, situations (4),
   - shall contain learning materials and guidelines for teachers (3).
6. Commonalities across differences

Lena Lindenskov

This handbook shows how being numerate is relevant for all, and it shows examples of good coaching practices and theoretical thoughts about doing and learning mathematics in actual real life situations. The examples are approved through experiments in the years 2004 - 2007 in Denmark, Hungary, Lithuania, the Netherlands, Norway, Slovenia and Spain.

We have been guided by the following research questions in our endeavour to explore mathematics in action:
1. Why do adults come back to school?
2. What do they want to learn?
3. How do they learn best?

1. Why do we teach adults in adult learning centers?
2. What do we teach?
3. What can be the meaning of an adult learning centre for learning in practice, in out-of-school situations?
4. How can we arrange a situation in which the adult learning center can be a center for transfer of learning in a school situation to learning in an out-of-school situation?

1. How can we challenge adults to learn more about mathematics in an out-of-school situation?
2. What role can an adult learning center play in supporting and coaching learning mathematics in out-of-school situations?

We experienced that conditions for adults learning mathematics differ a lot across Europe. Courses for learning mathematics in adult learning can be organised in many different ways, and teachers can have quite different background education and knowledge. We also experienced that Mathematics in Action established forms a commonality across differences. Despite organisational differences and differences in educational culture and tradition, we succeeded in finding commonalities through the exploration of the research questions.

One commonality is that teachers in mathematics in adult education wish to be informed and trained in how adults learn and use mathematics in out-of-school situations. A second commonality across differences is that adult students in the participating countries enjoyed the experiments. Teachers and learners describe the MiA alternative ways of learning and teaching mathematics as motivating and effective. A third commonality across differences is that teachers share fundamental values of adult education, adult learners and mathematics in life. A fourth commonality is that the combination of examples of good practice and relevant theoretical input inspires the teachers. The combination provides the teachers food for thought, it motivates for reflection in own practices, and it invites teachers to adapt the examples into own social contexts, suitable for local needs and opportunities, and suitable appropriate for local teachers and learners. Only theory is barren; only examples coming from different practices is also barren. In combination they can fertilise our...
view of education, learners and mathematics, where education is seen as a way to include people in society, and where adults are seen as wanting to understand the world around them, and why things are as they are.

Many adults experienced mathematics as little motivating and as very difficult. So it is a goal in itself to increase adult learners’ motivation by making learning more attractive and relevant. With the MiA ideas it is our hope to achieve a raise in quality of learning and teaching of mathematics in adult education in the EU countries, to support participation and to increase success rates of adult learners.

Target groups are teachers in adult general and vocational adult education and teacher trainers. They are the readers and users of the handbook. Only reading a book – even with many examples and narratives - is not a very effective method. That is why we have also developed structure and materials for teacher training workshops where teachers together with other teachers can try out models and examples for how to deal with several real-life situations, such as learning and doing at the workplace, at home, and in societal life.

The MTWs also devote time to discuss and reflect together on learning and teaching mathematics. MTWs do not offer a new full programme. They provide alternative ways of learning and teaching mathematics to be used alongside usual teaching methods, as well outside as inside usual classroom environments.. In this way we recognise teachers as important agents in adapting and putting MiA ideas at the right place. Time for discussion, experimenting and reflection is a must.

Although target groups are experienced teachers in professional development, we think the materials and the MTWs are also relevant for pre-service teacher education. All MiA partners will be eager to arrange MiA teacher training workshops, MTWs and discuss how MiA ideas can be adapted to more local situations and contexts.

Lena Lindenskov
DPU, University school of Education – Aarhus University
Tuborgvej 164
2400 Copenhagen
Project Leader MiA
<lenali@dpu.dk>
Appendices

Appendix 1: IALS results
Appendix 2: DeSeCo Key competences
Appendix 3: Paul Ernest: Transfer of information
Appendix 4: Guidelines for the 6 steps
Appendix 5: Pre-Questionnaire
Appendix 6: Post-Questionnaire
Appendix 7: Workshop Leaders evaluation
Appendix 1: Results IALS Survey - Quantitative Literacy (1996)


<table>
<thead>
<tr>
<th>Countries</th>
<th>Level 1-2</th>
<th>Level 3</th>
<th>Level 4-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sweden</td>
<td>25.2</td>
<td>39</td>
<td>35.8</td>
</tr>
<tr>
<td>Germany</td>
<td>33.3</td>
<td>43.2</td>
<td>23.5</td>
</tr>
<tr>
<td>Netherlands</td>
<td>35.8</td>
<td>44.3</td>
<td>19.9</td>
</tr>
<tr>
<td>Switzerland (Fr)</td>
<td>37.4</td>
<td>42.2</td>
<td>20.4</td>
</tr>
<tr>
<td>Belgium (Fl)</td>
<td>39.7</td>
<td>37.8</td>
<td>22.6</td>
</tr>
<tr>
<td>Switzerland (G)</td>
<td>40.4</td>
<td>40.7</td>
<td>19</td>
</tr>
<tr>
<td>Canada</td>
<td>43</td>
<td>34.8</td>
<td>22.2</td>
</tr>
<tr>
<td>Australia</td>
<td>43.3</td>
<td>37.7</td>
<td>19.1</td>
</tr>
<tr>
<td>USA</td>
<td>46.3</td>
<td>31.3</td>
<td>22.5</td>
</tr>
<tr>
<td>New Zealand</td>
<td>49.3</td>
<td>33.4</td>
<td>17.2</td>
</tr>
<tr>
<td>UK</td>
<td>51</td>
<td>30.4</td>
<td>18.6</td>
</tr>
<tr>
<td>Ireland</td>
<td>53.1</td>
<td>30.7</td>
<td>16.2</td>
</tr>
<tr>
<td>Poland</td>
<td>69.2</td>
<td>23.9</td>
<td>6.8</td>
</tr>
</tbody>
</table>


For the partners in the MiA project the results for quantitative literacy were as follows:

<table>
<thead>
<tr>
<th></th>
<th>(S) IALS level 1-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Netherlands</td>
<td>35.8%</td>
</tr>
<tr>
<td>Denmark</td>
<td>27.7%</td>
</tr>
<tr>
<td>Norway</td>
<td>29.0%</td>
</tr>
<tr>
<td>Slovenia</td>
<td>65.4%</td>
</tr>
<tr>
<td>Hungary</td>
<td>52.1%</td>
</tr>
</tbody>
</table>


**Level 1** indicates persons with very poor skills, where the individual may, for example, be unable to determine the correct amount of medicine to give a child from information printed on the package.

**Level 2** respondents can deal only with material that is simple, clearly laid out, and in which the tasks involved are not too complex. It denotes a weak level of skill, but more hidden than Level 1. It identifies people who can read, but test poorly. They may have developed coping skills to manage everyday literacy demands, but their low level of proficiency makes it difficult for them to face novel demands, such as learning new job skills.
Appendix 2: Key Competencies as formulated in DeSeCo.

Key Competences for Lifelong Learning  
- A European Reference Framework

Introduction  
This Framework sets out the eight key competences:

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Communication in the mother tongue;</td>
</tr>
<tr>
<td>2.</td>
<td>Communication in the foreign languages;</td>
</tr>
<tr>
<td>3.</td>
<td>Mathematical competence and basic competences in science and technology;</td>
</tr>
<tr>
<td>4.</td>
<td>Digital competence;</td>
</tr>
<tr>
<td>5.</td>
<td>Learning to learn;</td>
</tr>
<tr>
<td>6.</td>
<td>Interpersonal, intercultural and social competences and civic competence;</td>
</tr>
<tr>
<td>7.</td>
<td>Entrepreneurship; and</td>
</tr>
<tr>
<td>8.</td>
<td>Cultural expression.</td>
</tr>
</tbody>
</table>

Competences are defined here as a combination of knowledge, skills and attitudes appropriate to the context. Key competences are those which all individuals need for personal fulfilment and development, active citizenship, social inclusion and employment. By the end of compulsory education young people should have developed the key competences to a level that equips them for adult life, and they should be further developed, maintained and updated as part of lifelong learning.

Many of the competences overlap and interlock: aspects essential to one domain will support competence in another. Competence in the fundamental basic skills of language, literacy, numeracy and ICT is an essential foundation for learning, and learning to learn supports all learning activities. There are a number of themes that are applied throughout the Framework: critical thinking, creativity, initiative taking, problem solving, risk assessment, decision taking, and managing feelings constructively play a role in all eight key competences.
Key Competences

1. Communication in the mother tongue

**Definition:** Communication in the mother tongue is the ability to express and interpret thoughts, feelings and facts in both oral and written form (listening, speaking, reading and writing), and to interact linguistically in an appropriate way in the full range of societal and cultural contexts — education and training, work, home and leisure.

**Essential knowledge, skills and attitudes related to this competence**

Communication in the mother tongue requires an individual to have **knowledge** of basic vocabulary, functional grammar and the functions of language. It includes an awareness of the main types of verbal interaction, a range of literary and non-literary texts, the main features of different styles and registers of language, and the variability of language and communication in different contexts.

Individuals should have the **skills** to communicate in oral and written forms in a variety of communicative situations and to monitor and adapt their own communication to the requirements of the situation. Competence also includes the abilities to write and read different types of texts, search, collect and process information, use aids, formulate and express one’s own arguments in a convincing way appropriate to the context.

A positive **attitude** towards communication in the mother tongue involves a disposition to critical and constructive dialogue, an appreciation of aesthetic qualities and a willingness to strive for them, and an interest in interaction with others.

2. Communication in foreign languages

**Definition:** Communication in foreign languages broadly shares the main skill dimensions of communication in the mother tongue: it is based on the ability to understand, express and interpret thoughts, feelings and facts in both oral and written form (listening, speaking, reading and writing) in an appropriate range of societal contexts — work, home, leisure, education and training — according to one’s wants or needs. Communication in foreign languages also calls for skills such as mediation and intercultural understanding. An individual’s level of proficiency will vary between the four dimensions, different languages and according to their background, environment and needs/interests.

**Essential knowledge, skills and attitudes related to this competence**

Competence in additional or foreign languages requires **knowledge** of vocabulary and functional grammar and an awareness of the main types of verbal interaction and registers of language. Knowledge of societal conventions, and the cultural aspect and variability of languages is important.

**Essential skills** consist of the ability to understand spoken messages, to initiate, sustain and conclude conversations and to read and understand texts appropriate to the individual’s needs.

Individuals should also be able to use aids appropriately, and learn languages also informally as part of lifelong learning.

A positive **attitude** involves the appreciation of cultural differences and diversity, and an interest and curiosity in languages and intercultural communication.
3. Mathematical competence and basic competences in science and technology

**Definition:**
A. Mathematical competence is the ability to use addition, subtraction, multiplication, division and ratios in mental and written computation to solve a range of problems in everyday situations. The emphasis is on process and activity, as well as knowledge. Mathematical competence involves - to different degrees - the ability and willingness to use mathematical modes of thought (logical and spatial thinking) and presentation (formulas, models, constructs, graphs/charts).
B. Scientific competence refers to the ability and willingness to use the body of knowledge and methodology employed to explain the natural world, in order to identify questions and to draw evidence-based conclusions. Competence in technology is viewed as the application of that knowledge and methodology in response to perceived human wants or needs. Both areas of this competence involve an understanding of the changes caused by human activity and responsibility as an individual citizen.

**Essential knowledge, skills and attitudes related to the competence**

A. Necessary **knowledge** in mathematics includes a sound knowledge of numbers, measures and structures, basic operations and basic mathematical presentations, an understanding of mathematical terms and concepts, and of the questions to which mathematics can offer answers.
An individual should have the **skills** to apply basic mathematical principles and processes in everyday contexts at home and work, and to follow and assess chains of arguments. They should be able to reason mathematically, understand mathematical proof and communicate in mathematical language, and to use appropriate aids.
A positive **attitude** in mathematics is based on the respect of truth and willingness to look for reasons and to assess their validity.

B. For **science and technology**, the essential **knowledge** comprises the basic principles of the natural world, fundamental scientific concepts and principles, and technology and technological products and processes. Individuals should have an understanding of the advances, limits and risks of scientific applications and technology in societies at large (in relation to decision-making, values, moral questions, culture etc), both in specific areas of science such as medicine, and also an understanding of the impact of science and technology on the natural world, and the implications for sustainable development.
**Skills** include the ability to use and manipulate technological tools and machines as well as scientific data to achieve a goal or to reach a decision or conclusion. Individuals should also be able to recognise the essential features of scientific inquiry and have the ability to communicate the conclusions and reasoning that led to them.
Competence includes an **attitude** of critical appreciation and curiosity, an interest in ethical issues and respect for both safety and sustainability - in particular as regards scientific and technological progress in relation to oneself, family, community and global issues.
4. Digital competence

**Definition:** Digital competence involves the confident and critical use of Information Society Technology (IST) for work, leisure and communication. It is underpinned by basic skills in ICT: the use of computers to retrieve, assess, store, produce, present and exchange information, and to communicate and participate in collaborative networks via the Internet.

**Essential knowledge, skills and attitudes related to the competence**

Digital competence requires a sound understanding and knowledge of the nature, role and opportunities of IST in everyday contexts: in personal and social life as well as at work. This includes main computer applications such as word processing, spreadsheets, databases, information storage and management, and an understanding of the opportunities of Internet and communication via electronic media (e-mail, network tools) for leisure, information sharing and collaborative networking. Individuals should also understand how IST can support creativity and innovation, and be aware of issues around the validity and reliability of information available and the ethical principles of in the interactive use of IST.

**Skills** needed include: the ability to search, collect and process information and use it in a critical and systematic way, assessing relevance and distinguishing real from virtual while recognising the links. Individuals should have skills to use tools to produce, present and understand complex information and the ability to access, search and use internet-based services; they should also be able use IST to support critical thinking, creativity, and innovation.

Use of IST requires a critical and reflective attitude towards available information and a responsible use of the interactive media; an interest in engaging in communities and networks for cultural, social and/or professional purposes also supports competence.

5. Learning to learn

**Definition:** ‘Learning to learn’ is the ability to pursue and persist in learning. Individuals should be able to organise their own learning, including through effective management of time and information, both individually and in groups. Competence includes awareness of one’s learning process and needs, identifying available opportunities, and the ability to handle obstacles in order to learn successfully. It means gaining, processing and assimilating new knowledge and skills as well as seeking and making use of guidance. Learning to learn engages learners to build on prior learning and life experiences in order to use and apply knowledge and skills in a variety of contexts – at home, at work, in education and training. Motivation and confidence are crucial to an individual’s competence.

**Essential knowledge, skills and attitudes related to the competence**

Where learning is directed towards particular work or career goals, an individual should have knowledge of the competences, knowledge, skills and qualifications required. In all cases, learning to learn requires an individual to know and understand their preferred learning strategies, the strengths and weaknesses of their skills and qualifications, and to be able to search the education and training opportunities and guidance/support available to them. Learning to learn skills require firstly the acquisition of the fundamental basic skills such as literacy and numeracy that necessary for further learning. Building on this, an individual should be able to access, gain, process and assimilate new knowledge and skills. This requires
effective management of one’s learning, career and work patterns, and in particular the ability to persevere with learning, to concentrate on extended periods and to reflect critically on the purposes and aims of learning. Individuals should be able to dedicate time to learning autonomously and with self-discipline, but also to work collaboratively as part of the learning process, draw the benefits from a heterogeneous group, and to share what they have learnt. They should be able to evaluate their own work, and to seek advice, information and support when appropriate.

A positive attitude includes the motivation and confidence to pursue and succeed at learning throughout one’s life. A problem-solving attitude supports both learning and an individual’s ability to handle obstacles and change. The desire to apply prior learning and life experiences and the curiosity to look for opportunities to learn and apply learning in a variety of life-wide contexts are essential elements of a positive attitude.

6. Interpersonal, intercultural and social competences, civic competence

**Definition:** These competences cover all forms of behaviour that equip individuals to participate in an effective and constructive way in social and working life, and particularly in increasingly diverse societies, and to resolve conflict where necessary. Civic competence equips individuals to fully participate in civic life, based on knowledge of social and political concepts and structures and a commitment to active and democratic participation.

**Essential knowledge, skills and attitudes related to the competence**

A. Personal and social well-being requires an understanding of good physical and mental health as a resource for oneself and one’s family and knowledge of how to build and maintain it through a healthy lifestyle. For successful interpersonal and social participation it is essential to understand the codes of conduct and manners generally accepted in different societies and to be aware of basic concepts relating to individuals, groups, society and culture. As regards identity, understanding the multi-cultural dimension of European societies and that it consists of national cultural identity in interaction with one of Europe and the rest of the world is essential.

**Skills** to communicate constructively, express and understand different viewpoints, negotiate with the ability to create confidence, and feel empathy are the core of this competence. Individuals should be able to cope with stress and frustration and to express it in a constructive way and should also distinguish between the personal and professional spheres. As regards **attitudes**, the competence is based on assertiveness and integrity. Individuals should have an interest in intercultural communication, value diversity and respect others, and be prepared both to overcome prejudices and to compromise.

B. Civic competence is based on knowledge of the concepts of democracy, citizenship, and civil rights, including how they are expressed in international declarations and applied by various institutions at the local, regional, national, European and international levels. Knowledge of main events, trends and agents of change in national, European and world history and present, with a specific view on European diversity is essential, as is knowledge of the aims, values and policies of social and political movements.

**Skills** relate to the ability to engage effectively with others in the public domain, display solidarity and interest in solving problems affecting the local and wider community. It involves critical and creative reflection and constructive participation in community/
neighbourhood activities as well as decision-making at all levels from local to national and 
European level, in particular by voting. 
Respect of human rights and equality as a basis for democracy, appreciation and 
understanding of differences between value systems of different religious or ethnic groups lay 
the foundations to a positive **attitude**. It comprises also the display of a sense of belonging to 
one’s locality, country, EU and Europe in general and (one’s part of) the world and the 
willingness to participate in democratic decision making at all levels. Constructive 
participation also involves civic activities, support for social diversity and cohesion and 
sustainable development, and a readiness to respect the values and privacy of others with a 
propensity to react against anti-social behaviour.

### 7. Entrepreneurship

| **Definition:** Entrepreneurship refers to an individual’s ability to turn ideas into action. It 
| includes creativity, innovation and risk taking, as well as the ability to plan and manage 
| projects in order to achieve objectives. This supports everyone in day to day life at home and 
| in society, employees in being aware of the context of their work and being able to seize 
| opportunities, and is a foundation for entrepreneurs establishing social or commercial activity.

**Essential knowledge, skills and attitudes related to the competence**

- **Necessary knowledge** includes available opportunities for personal, professional and/or 
  business activities, including ‘bigger picture’ issues that provide the context in which people 
  live and work, such as a broad understanding of the workings of the economy, and the 
  opportunities and challenges facing an employer or organisation. Individuals should also be 
  aware of the ethical position of enterprises, and how they can be a force for good for example 
  through fair trade or through social enterprise.

- **Skills** relate to proactive project management (involving skills such as planning, organising, 
  analysing, communicating, managing, de-briefing and evaluating and recording), and the 
  ability to work both as an individual and collaboratively in teams. The judgement to identify 
  one’s strengths and weaknesses, and to assess and take risks as and when warranted is 
  essential.

- An entrepreneurial **attitude** is characterised by initiative, independence and innovation in 
  personal and social life, as much as at work. It also includes motivation and determination to 
  meet objectives, whether personal goals or aims held in common with others, and/or at work.
8. Cultural expression

**Definition:** Appreciation of the importance of the creative expression of ideas, experiences and emotions in a range of media, including music, performing arts, literature, and the visual arts.

**Essential knowledge, skills and attitudes related to the competence**

Cultural **knowledge** includes a basic knowledge of major cultural works, including popular contemporary culture as an important part of human history in the contexts of national and European cultural heritage and their place in the world. It is essential to understand the cultural and linguistic diversity of Europe (and European countries) as well as the evolution of popular taste and the importance of aesthetic factors in daily life.

**Skills** relate to both appreciation and expression: self-expression through the variety of the media with individuals’ innate capacities and appreciation and enjoyment of works of art and performances. Skills include also the ability to relate one’s own creative and expressive points of views to the opinions of others and to identify and realise economic opportunities in cultural activity.

A strong sense of identity is the basis for respect and open **attitude** to diversity of cultural expression. A positive attitude also covers creativity, and the willingness to cultivate aesthetic capacity through artistic self-expression and interest in cultural life.
Appendix 3: Paul Ernest’s view on transfer of knowledge

1. Applications Perspective:

According to the applications of perspective, modeling builds a link with the applications context. The claim of this perspective is that the problem of transfer is thus overcome. The ‘real world’ context of the application and the academic or school mathematics context are in a dialogical relation which build permanent bridges between them, connecting both. First of all, representations from the context of application provide the basis for generating concepts methods and problems in the academic or school mathematics context, via abstraction and generalization. So there is a flow from the context of application to that of schooling. Second, there is a flow in the other direction. Abstract mathematical knowledge, concepts, skills and models in the school context are used in applications and verified in the context of application. Ultimately, when the knowledgeable user of mathematics is immersed in applications, the academic/applications concept becomes irrelevant. For mathematics models can be formulated in either. The important difference becomes that between the abstract level of models and the ‘concrete world’ level of empirical problems, solutions and data. The applications perspective on transfer and inter-contextual relations is illustrated in Figure 1.

Figure 1: The applications perspective on transfer and inter-contextual relations in mathematics

<table>
<thead>
<tr>
<th>‘Real world’ context</th>
<th>Generation</th>
<th>Academic/school maths context</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>→</td>
<td></td>
</tr>
<tr>
<td>abstraction</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concrete and applied maths problems and applications</td>
<td>Application</td>
<td>abstract mathematical knowledge concepts and skills</td>
</tr>
<tr>
<td></td>
<td>←</td>
<td></td>
</tr>
<tr>
<td>verification</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. Cognitivist Perspective

According to the cognitivist perspective explicit mathematical knowledge learned in school maths contexts is transferable to external uses in the ‘real world’ contexts of numeracy and mathematics. Explicitly learned school mathematics, including symbol systems and computational algorithms, as disembedded knowledge, is applicable to mathematically-susceptible tasks originating in domestic, popular, work and other external contexts. This facility depends on the ability to identify and then work mathematical tasks located in these external situations. These are external task representations together with some incidental features associated with them to help future identification. The cognitivist perspective on transfer and inter-contextual relations for mathematics is illustrated in Figure 2.

Figure 2: The cognitivist perspective on transfer and inter-contextual relations for mathematics
3. Problem Solving Perspective (constructivist):

The problem solving perspectives views the most important knowledge for transfer as tacit personal knowledge, namely problem solving strategies and heuristics. This knowledge is acquired primarily from solving non-routine problems in the school context, plus from seeing teachers and other showing solution problems for particular situations. The key feature issue of the problem solving perspectives is that the most significant inter-contextually transferable skills are the personally acquired, personally transportable heuristics and higher-level skills. An important current issue concerning transfer of skills which fits under this perspective is that of ‘personal transferable skills’. (MvG: comparable with competencies)

See also the list of personal transferable skills below.

Fig 3. The problem solving perspective on transfer and inter-contextual relations in mathematics

Ernest added the following list of personal transferable skills:

- self-management
- learning skills
- communication skills
- teamwork skills
- problem-solving
- data-handling skills
4. Situated Cognition Perspective (social theorists)

The situated cognition perspective is that mathematical knowledge is partly situated and some of it cannot be divorced from the context of origin ands deployment.

This picture depicts a number of separate contexts. We have the school mathematics context and some other contexts of which sample are shown in Figure 4, including the domestic and popular context(s) of numeracy and maths use, the industrial and work context of maths applications, the academic university maths context. These have different aims, roles, functions and practices, and there is discussion of the problem of transfer from one of these to another.

Fig.4: The situated cognition perspective on transfer and inter-contextual relations.
Appendix 4: **Guidelines for the 6 steps**

Learning/coaching/supporting/challenging/facilitating learning experiments
How do people solve mathematical problems in real life situations and how do they learn from that?
How to find out and how to become more knowledgeable as a teacher?
Report of the learning experiment in the following steps:

**First:**
Who are the adult participants regarding gender, age, ethnicity, how many?
Names of the teachers
Dates of the experiment and how long did it take?
Where did the learning/teaching experiment take place. (in an educational institution, or in a different situation, as part of a course, or ….. etc.)
Please describe details

**Write your text here …………..**

**1) bring the learner in a potential mathematical situation , e.g. sales**
The teacher knows that they may encounter a mathematical problem in the situation
- by bringing the learner in an actual authentic situation, e.g. in department store or street market
- by asking them to tell a story
- by bringing and showing something with a discount price (either the learners or the teachers)
e.g. show a coat priced 150 euro. With a label: 15% off

For the analysis of the experiment we ask the teacher to describe the context situation and described what happened.

**Write your text here …………..**

**2) identify problems in the situation**
- focus or zoom in on mathematical problems, e.g. the learners says: “I don’t how to compute the new price. I just pay the amount at the cashier desk they ask me to pay”

For the analysis of the experiment we ask the teacher to describe what the learner identified as a problem

**Write your text here …………..**
3) **planning the problem solving procedure:**
- how do you think you can solve the problem?
learners may find all kinds of informal and formal problem solving procedures.

Teacher’s task is to interact with the learners and try to discover what learners know and can do and what they don’t know or do wrong, e.g. the learner states that 10% is always 10 euro off.

For the analysis of the experiment we ask the teacher to describe what kinds of conceptions and misconceptions do learner may have. What kind of computations do they apply?

*Write your text here …………*

4) **Problem solving procedure:**
at this point the learning process can start, e.g. by discussions among learners (interaction)
Try to connect with the learners pre-knowledge and good conceptions.
e.g. the learner knows that 50% is half. How would you go on?

For the analysis of the experiment we ask the teacher to describe what the learner knows concerning the topic. What kind of (mis) conceptions the learner has and what kind of computations the learner applies.

*Write your text here …………*

5) **check the result:**
the learners can explain why their answer or solution is correct or not
Can the learners explain why their answer or solution is correct or not?

For the analysis of the experiment we ask the teacher to describe whether the learn can explain his/her reasoning and computations.

*Write your text here …………*

6) **review the process**
What did the learner learn?
The learners discuss what they learned

For the analysis of the experiment we ask the teacher to describe what the learner learned.

*Write your text here …………*

Teacher’s evaluation in the end:
For the analysis of the experiment we ask the teacher to describe what he/she learned as a teacher.

*Write your text here …………*
### Questionnaire sent out to the participants before the conference

<table>
<thead>
<tr>
<th>Question</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>Name:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Work place:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Background education:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>On what level do you teach?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>For how many years have you taught on this level?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Do you have any experience in adult education? (put in x)</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>If yes, for how long? (put in x): 1-5 years 6-10 years 11-20 years over 20 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Have you taught maths to adults? (put in x)</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>For how long? (put in x): 1-5 years 6-10 years 11-20 years over 20 years</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Do you have any specific courses for working with adults? (put in x)</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>If yes, what kind of course?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Do you use existing materials? (put in x)</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Do you use instruction materials you have developed yourself? (put in x)</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>If yes, in what kind of way?</td>
<td></td>
<td></td>
</tr>
<tr>
<td>What kind of expectations do you have concerning this course?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Appendix 6. Post-Questionnaire

(Mathematics in Action - Workshop Teachers QUESTIONNAIRE

Please, write your opinion about the following statements and answer the questions. Your answers will be used for evaluation of the workshop. Individual answers and identity will remain confidential.

1. The information about the workshop I received beforehand was:
   A – excellent  B - good  C - reasonable  D - poor  E- unsatisfactory

2. The information from the questionnaire I filled in beforehand was used in the workshop.
   A - true  B - partly true  C - no opinion  D - partly not true  E - not true

3. The workshop met my expectations.
   A - true  B - partly true  C - no opinion  D - partly not true  E - not true

4. The balance between theory and practical work was:
   A – excellent  B - good  C - reasonable  D - poor  E- unsatisfactory

5. I was able to express my opinions freely.
   A - true  B - partly true  C - no opinion  D - partly not true  E - not true

6. The materials I received during the workshop were:
   A – excellent  B - good  C - reasonable  D - poor  E- unsatisfactory

7. The methods the workshop leader used were:
   A – excellent  B - good  C - reasonable  D - poor  E- unsatisfactory

8. Time was used effectively.
   A – true  B - partly true  C - no opinion  D - partly not true
9. What was the most interesting/useful for your work?

________________________________________________________________________
________________________________________________________________________

10. Do you have any suggestions for improvement?

________________________________________________________________________
________________________________________________________________________

11. Are you planning to use the six steps or seven principles in your teaching practice?
   A – yes. Please, explain:
   ________________________________________________________________
   ________________________________________________________________
   B – no. Please, explain:
   ________________________________________________________________
   ________________________________________________________________

12. Do you think these methods and principles are useful also in (adult) education in general?
   A – yes. Please, explain:
   ________________________________________________________________
   ________________________________________________________________
   B – no. Please, explain:
   ________________________________________________________________
   ________________________________________________________________

13. Are you interested in further training related to these themes?
   A - yes    B – no

14. Would you recommend this workshop to your colleagues?
   A - yes    B – no

15. Please, explain in what way you expect the MiA teacher’s handbook could be the most useful for your work?
Appendix  7 Evaluation for workshop leaders

<table>
<thead>
<tr>
<th>Country:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Title:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Name/s of the leader/s:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Date:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Duration in hours:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Number of participants:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
</tr>
</tbody>
</table>

Short description of the participants (from the application questionnaire/form; age, gender, degree, teaching experiences, where they came from ...):

Appendix (programme of the workshop, materials):
II.- Summary of the evaluation questionnaire
Which of the *research questions* generated more discussion in the group? And what did you find interesting to put in the handbook?

In what way do you think the reflection you had in the workshop on Greeno can be useful for teachers’ teaching/coaching practice?

How did you use the *seven principles* of dialogical learning during the workshop?

How did you use the method of *six steps* during the workshop?

Please, write three positive and three negative aspects of the workshop:

<table>
<thead>
<tr>
<th>+</th>
<th>-</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
</tbody>
</table>